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COMPUTER BASED SUBSIDENCE PREDICTION TECHNIQUES: A LITERATURE REVIEW AND EVALUATION OF THEIR APPLICATION TO UNDERGROUND CANADIAN COAL MINING

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EXECUTIVE SUMMARY

An understanding of the mechanisms involved in surface subsidence over underground coal mining (and the ability to reliably predict it) is essential for mining safety and environmental protection. This study forms one part of a major subsidence research program which has been initiated by the Cape Breton Coal Research Laboratory of the Canada Centre for Mineral and Energy Technology (CANMET) for the Sydney coal field, Nova Scotia.

A review and assessment of the empirical and analytical methods which can be used to predict surface subsidence over underground coal mining has been undertaken. The empirical methods include the U.K. National Coal Board type curves, profile functions, influence functions, incremental design and stochastic methods. The analytical methods which were reviewed include closed-form elastic solutions, the finite difference method, the finite element method, the boundary integral method, and a mechanistic model.

A survey of accessible and documented computer programs which span the empirical and analytical subsidence prediction methods was undertaken. The strengths and limitations of the programs are discussed. Six specific programs (ZONE, BERRY.3D, GEOROC, SUBSID-B, ADINA, MSEAMS) were selected and used to analyze a hypothetical case history which is typical of the Sydney coalfield. The results indicate that there is a major difference between subsidence values obtained by empirical methods and those produced by analytical means. Each group yielded results which are internally consistent, but the latter yielded much lower vertical displacements than the former method. These differences may be due to the fact that the input data are not representative of field values and/or to the fact that there are other mechanisms (such as bed separation, roof caving, joint slip) besides elastic strata bending involved in the subsidence process which act to decrease the overall stiffness of the overburden and seam model. At present there is no analytical method which can accurately model these mechanisms nor is it known which methods are significant. A few additional analyses were performed and these suggest that the field deformation modulus may be a useful parameter which can implicitly incorporate the effects of some of the subsidence mechanisms.

An assessment of future trends indicates that the technical capability of subsidence analysis will increase in the next decade in parallel with the growth in computing power. It is felt that the finite element method and hybrid methods offer the most potential.

It is recommended that a long term program be established to compile a comprehensive data file of subsidence case histories. Research is also required to define the mechanisms involved in the subsidence process. This can be done by computer analysis using the identification method and/or by direct field experiments. It is also recommended that CANMET acquire a few subsidence programs and perform analyses of the Sydney coalfield mines. It is possible that relevant data for empirical analysis may be obtained from coal mines in the Appalachians of the U.S.A. with similar geology.

COMPTE-RENDU DE LA DIRECTION

La sécurité dans les mines de charbon et la protection de l'environnement exigent une compréhension des mécanismes intervenant dans l'affaissement de la surface des mines souterraines (et la capacité de prévoir de façon fiable cet affaissement). Cette étude fait partie d'un vaste programme de recherches sur l'affaissement qui a été entrepris par le Laboratoire de recherches sur le charbon du Cap Breton, organisme du Centre canadien de la technologie des minéraux et de l'énergie (CANMET), dans le bassin houiller de Sydney en Nouvelle-Écosse.

Ce rapport passe en revue et évalue les méthodes empiriques et analytiques qui peuvent servir à prédire l'affaissement de la surface au-dessus des mines souterraines de charbon. Les méthodes empiriques comprennent les courbes de même type que celles de la Commission nationale du charbon du Royaume-Uni (U.K. Nat. Coal Board), les fonctions de profil, les fonctions d'influence, l'étude incrémentielle et les méthodes stochastiques. Les méthodes analytiques examinées comprennent les solutions élastiques de forme fermée, la méthode des différences finies, la méthode des éléments finis, la méthode intégrale limite, la méthode des modèles mécanistes et celle d'identification hybride.

Un examen des programmes d'ordinateur faciles d'accès et documentés qui couvrent les méthodes empiriques et analytiques de prédiction de l'affaissement fut entrepris. Six programmes spécifiques (ZONE, BERRY.3D, GEOROC, SUBSID-B, ADINA, MSEAMS) ont été choisis et utilisés pour l'analyse d'un cas hypothétique typique du bassin houiller de Sydney. Les résultats indiquent qu'il y a une différence importante entre les valeurs de l'affaissement obtenues par les méthodes empiriques et celles obtenues par les moyens analytiques. À l'intérieur de chaque groupe, les résultats sont consistents mais les moyens analytiques donnent (de façon non réaliste) des déplacements verticaux plus bas que ceux obtenus par les méthodes empiriques. Ces résultats indiquent qu'il y a des mécanismes autres que le fléchissement des couches élastiques (tels la séparation des lits, l'affaissement du toit, le glissement des joints) qui entrent en jeu dans le processus d'affaissement qui provoque une diminution de la rigidité globale des terrains de couverture et du gisement. À l'heure actuelle, il n'existe

aucune méthode analytique qui puisse offrir un modèle précis de ces mécanismes et de plus on ignore quelles méthodes sont significatives. Quelques analyses supplémentaires effectuées suggèrent que le module de déformation serait un paramètre utile qui pourrait inclure les effets de quelques mécanismes de l'affaissement.

Une projection des tendances actuelles indique qu'au cours des dix prochaines années, la capacité technique d'analyse de l'affaissement augmentera parallèlement à la puissance de calcul des ordinateurs. On croit que la méthode des éléments finis et celle d'identification offrent le plus de possibilités.

Il est suggéré qu'un programme à long terme soit mis sur pied pour dresser un fichier détaillé de cas d'affaissement. Une recherche qui aurait pour but la définition des mécanismes faisant partie du processus d'affaissement est également nécessaire. Cette recherche peut se faire par analyse sur ordinateurs en utilisant la méthode d'identification et/ou par des expériences effectuées sur le terrain. Il est de plus suggéré que CANMET se procure quelques programmes d'affaissement et effectue des analyses sur les mines du bassin houiller de Sydney. Il est possible que des données pertinentes pour une analyse empirique soient obtenues des mines de charbon des Appalaches, aux États-Unis, qui auraient une structure géologique semblable.

1 INTRODUCTION

Coal mining beneath the sea has been carried out in Canada, Australia, Chile, Spain, Great Britain and Japan with some of the workings extending over 8 km from the shoreline. A number of incidents of flooding from sea water have been reported and in the latter two countries there has been considerable loss of life. Two incidents of flooding have been reported in Canada, both in Cape Breton mines (Engineers International, 1979). Guidelines for submarine coal mining have been presented by Babcock and Hooker (1977) while a comprehensive review of guidelines, criteria and hydrogeological and rock mechanics factors related to submarine coal mining has been compiled by Engineers International (1979). It is clear that accurate subsidence prediction and a sound understanding of the associated strata movements is very important for submarine coal mines, both for safety and economic reasons.

This study forms one part of a major subsidence research program (Aston, 1983) which has been initiated in the Sydney coalfield, Nova Scotia, by the Cape Breton Coal Research Laboratory of the Canada Centre for Mineral and Energy Technology (CANMET). As part of the overall program it is expected that field measurements of subsidence will become available from the coalfield, which is located in strata beneath the Atlantic Ocean off the coast of Cape Breton, Nova Scotia. However, due to the difficulty of making sub-sea measurements, this information may be insufficient for formulating empirically-based subsidence prediction methods. Thus, the use of mathematical modeling and/or semi-empirical prediction techniques must be considered.

The objective of the study which is described in this report is to identify, review and evaluate all the numerical modeling techniques which have been used for subsidence research and to assess their practical application for the prediction of subsidence over the coalfields of Cape Breton. An important part of the study is to analyze a hypothetical case history which is typical of the Sydney coalfield using a number of different numerical methods. Based on the analyses and the literature review,

recommendations are made regarding which methods are most appropriate for predicting subsidence and what methods might become available with further development.

The process of subsidence is extremely complex, and a proper analytical understanding of it is one of the major challenges in rock mechanics today. Figure 1-1 shows the generalized strata behaviour above a longwall panel. It is seen that any subsidence prediction method must include information about the mining geometry as well as the rock properties for the entire geologic column. In addition it must be able to model mechanisms such as caving, compaction of the caved zone (and backfill, if any), bending of strata, bed separation and formation of surface tensile cracks. Experience has shown that the subsidence process is time dependent. Table 1-1 summarizes the requirements for non-empirical prediction methods. Even with modern numerical analytical techniques and powerful computer hardware it is difficult to accurately model all these complexities. In addition it is almost impossible to obtain rock mass properties in sufficient detail for adequate modeling.

Subsidence theories were postulated as early as the beginning of the nineteenth century. An excellent review of the development of these theories to present-day time is presented by Shadbolt (1978). Current subsidence prediction techniques can be divided into two major categories: (i) empirical and (ii) analytical/mechanistic techniques. Empirical techniques are based on the experience gained from observation of subsided areas, while analytical/mechanistic techniques are based on the response of physical properties of the subsiding material to the mining geometry. Empirical techniques are most widely used in areas where years of observations have been recorded, and in some cases are claimed to be accurate to within 10%. Analytical methods are limited by their complexity and their requirement that the subsiding material's physical properties be known. It is fair to say that no single analytical method available today can model subsidence and incorporate all the requirements outlined in Table 1-1. The dramatic growth in computer technology (both processing speed and memory size) is having a serious impact on this situation, however, and the sophistication and applicability of computer-based analytical methods is increasing rapidly.

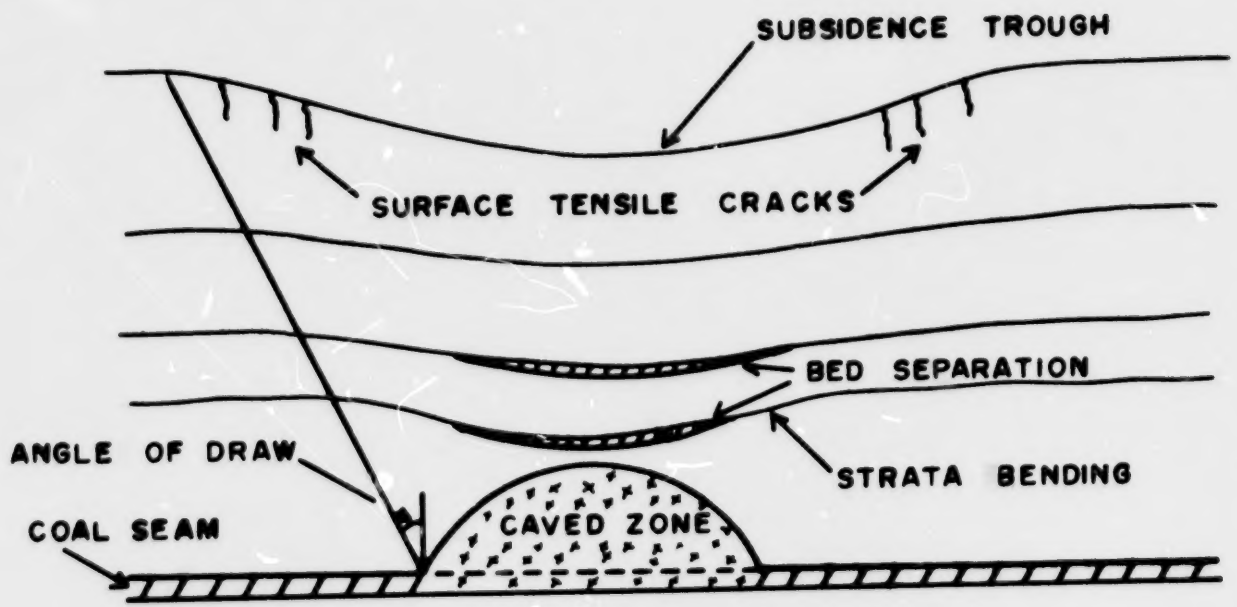

Figure 1-1. Generalized Strata Behavior Associated with Subsidence in Long Wall Coal Mining.

Table 1-1. Requirements for Analytical Subsidence Prediction Methods

(a) Input Data:

- Mine geometry
- Mining sequence
- Elastic and strength properties for entire geologic column*
- Mechanical properties of backfill and/or gob*
- Geometry and properties of discontinuities in rock mass*

(b) Mechanisms:

- Pillar deformation
- Caving
- Compaction of caved zone/backfill
- Strata bending
- Bed separation
- Formation of surface tensile cracks
- Slip along joints, fractures.

* Some of these may be non-linear and/or time dependent.

The remainder of this report reviews available subsidence prediction methods and is organized in the following fashion. Section 2 describes a number of empirical subsidence prediction methods. A detailed discussion of analytical subsidence prediction methods is presented in Section 3 along with a review of their limitations and applicability to the Cape Breton coalfield. Section 4 compiles information on specific subsidence computer programs which are publically available. A hypothetical mining situation, which includes stratigraphy typical of the Cape Breton region, is analyzed using six different computer programs and the results and discussions are presented in Section 5. Potential subsidence prediction methods which may be developed in the future are reviewed in Section 6. The conclusions and recommendations from this study are presented in Section 7.

A number of terms commonly used in subsidence analyses are defined in Table 1-2. The reader who is unfamiliar with the subsidence literature may find it useful to peruse this table prior to reading this report.

Table 1-2. Glossary of Terms Used in Subsidence Analysis

Angle of Draw (or Limit Angle).	The angle from the vertical of the line connecting the edge of the panel and the edge of the subsidence area.
Critical Area (Radius).	The area (radius) of a mined working that causes the complete subsidence of one point on the surface.
Sub-critical Area (Radius).	An area (radius) of working smaller than the critical area (radius). A point on the surface undergoes only partial subsidence.
Super-critical Area (Radius).	An area (radius) of working greater than a critical area. An area on the surface undergoes complete subsidence.
Transverse (Longitudinal) Profile.	A profile showing subsidence along a line drawn perpendicular (parallel) to the direction of advance of a longwall panel.
Subsidence Factor.	The ratio of complete subsidence to seam thickness.
Transition Point.	The point of transition between concave and convex curvature of a subsidence profile.
Break Angle.	The angle from the vertical of the line connecting the edge of the panel and the point of maximum tensile surface strain.

2 EMPIRICAL SUBSIDENCE PREDICTION METHODS

Subsidence prediction methods fall into one of the two categories: empirical methods and analytical/mechanistic methods. Empirical subsidence theories are based largely on experience, intuition and observations of ground movements without particular regard to the causes, or mechanisms, resulting in the subsidence. Predictions using empirical methods have been carried out successfully, particularly in the U.K. and European coalfields where data from field observations have been collected over long periods of time. Analytical/mechanistic methods have not found widespread acceptance because of the difficulty in determining the material properties of the overburden rocks and using a few simplified assumptions to simulate complex field problems. The three main empirical techniques are; the National Coal Board graphical method, the profile function method and the influence function method. The National Coal Board (NCB) approach represents a large collection of subsidence field data from U.K. longwall mines, summarized and analyzed through a series of dimensionless graphical relationships. The profile function method employs a mathematical function to match the observed subsidence profile and has been applied successfully in Poland, Hungary and the Soviet Union (Hood, Ewy and Riddle, 1983). Subsidence prediction using influence functions is based on the assumption that the relationship between the extraction of material and the resulting ground movement is linear (U.S. Bureau of Mines, 1983). Several case histories in the United States presented by Sutherland and Munson (1984) illustrate the capabilities of using the influence function method.

2.1 NATIONAL COAL BOARD GRAPHICAL METHOD

A graphical method of predicting the complete subsidence profile was developed by the U.K. National Coal Board and is described in a publication entitled the Subsidence Engineers Handbook (National Coal Board, 1965, 1975). The method of analysis was derived from an extensive field data base collected over many years from longwall mines. These data are summarized through a

series of non-dimensional graphs, and a typical example is shown in Figure 2-1.

Calculations are based on the ratio of the width of a working face to its depth and all principal factors including vertical subsidence and horizontal strain are related to this ratio. By using these graphs, subsidence for mines of similar design and geology, as given in Appendix 1 of the Handbook, may be predicted.

A basic requirement is that the mined excavation is rectangular. Other assumptions require that the total length of panel must be greater than 1.4 times the depth of seam and that panels must have no centre gates or other zones of special packing apart from those at the main and tail gates. Reduction of maximum subsidence is required when the length of a panel is less than 1.4 times the depth of the seam. Corrections to the basic subsidence profile to include the following effects are also given in the NCB handbook:

1. Effect of inclined seam.
2. Effect of sloping ground surface.
3. Effect of coal pillars.

Under appropriate conditions subsidence profiles predicted by this method are claimed to be within ten percent of the actual field measurements "in the great majority of cases." The National Coal Board (NCB) graphical method is undoubtedly the most commonly used subsidence prediction method in the world. It requires very few input parameters and allows for ready determination of subsidence in a straight-forward fashion. Empirical graphs for strain calculation are provided as well as correction factors due to coal pillars, inclined seams and sloping ground surface. Orchard (1969) calculated surface strains and subsidence due to multiple seam workings using the principal of superposition. However, all calculations are restricted to rectangular mine geometries. The theoretical limit angle is assumed to be 35° and significant deviation of subsidence profiles can occur if different angles of draw are encountered. No reference is made to the overburden stratigraphy, the distribution of

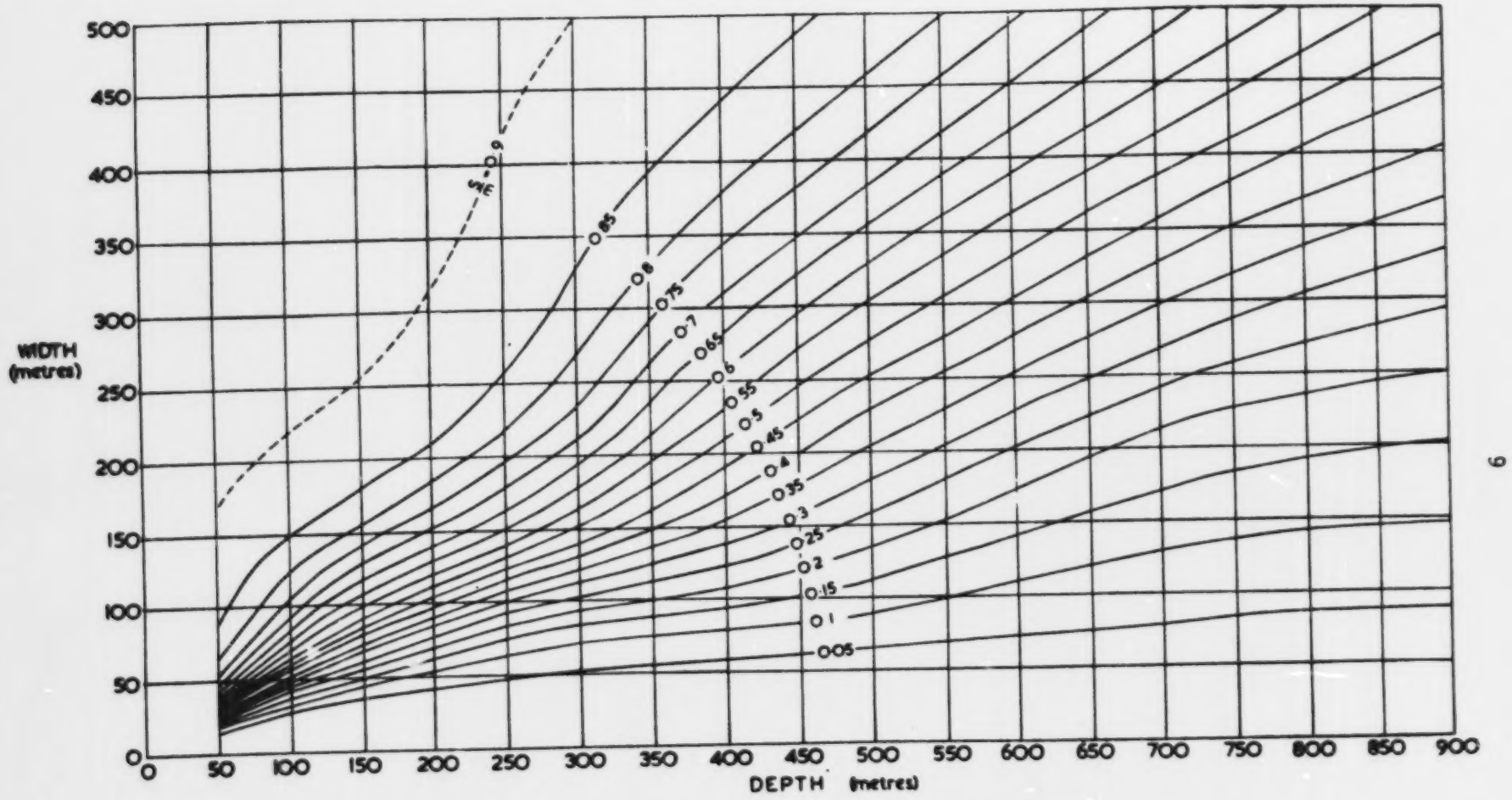

Figure 2-1. Typical National Coal Board (1975) Graph for Determining Subsidence over Long Wall Coal Mines (s = Maximum Subsidence, m = Seam Thickness).

joints and fractures, topographical site conditions, mechanical properties of rock and methods of mining, all of which influence the general subsidence development. The main concern with applying the NCB method is that the geology of Cape Breton is different from that in the U.K. and, thus, there is no reason that the NCB curves should be valid.

2.2 GRAPHICAL METHODS FOR ROOM AND PILLAR WORKINGS

The NCB graphs are applicable specifically to longwall mines but may be used for room and pillar mining in some instances where the pillars are robbed or are designed to crush out. Empirical graphs have been developed for the prediction of subsidence specifically over room and pillar workings by Wardell (1969), Abel and Lee (1980), and Mathur and Mikkilineni (1983). Their graphs are based on relatively limited observations of subsidence, and relate percentage subsidence to depth, extraction ratio, seam height and pillar width. Since these curves are based on very limited data and since little room and pillar mining is being planned for the Cape Breton coalfield, this method is not discussed further.

2.3 PROFILE FUNCTIONS

The profile function technique is based on curve-fitting procedures and employs a mathematical profile function to match observed subsidence data. Once a fit has been established, the function is used to predict subsidence for new mines in the same coal field or in similar geologic terrain. This method has been employed successfully in Poland, Hungary and the USSR.

Examples of early subsidence formulae are quoted by Grond (1953) based on the work of O'Donahue and Louis (1922). These functions were found to be unacceptable since they are purely a function of the depth and extracted seam height. However, as more subsidence information became available, a number of different types of mathematical functions over critical areas have been used. These functions are all of the following general form:

$$s(x) = S.f(B,x,c) \qquad (2.1)$$

where $s(x)$ = subsidence at point x

S = subsidence at the centre of the panel

B = parameter that controls the range of the function; for non-asymptotic functions B is the critical radius and is given by

$$B = h \tan a$$

where h = depth of seam

a = angle of draw

x = horizontal distance from centre of panel

c = a function or a constant or in some cases the distance between the point of half-maximum subsidence and the edge of the opening (See Figure 2-2).

The general shape of a subsidence profile function is shown in Figure 2-2. Examples of profile functions are listed by Brauner (1973) and given in Table 2-1. These functions are generally of an asymptotic or non-asymptotic shape. Also, each function is either symmetrical or anti-symmetrical about the inflection point. In most cases the inflection point is located at the point of one half the maximum subsidence. The location of the point of inflection must be determined from field measurements. The hyperbolic and error functions are asymptotic to s=0 and s=S and therefore theoretically full subsidence and zero subsidence do not exist in these curves.

Special considerations are required when there is subcritical subsidence. Special profile functions have been derived for this case (see Table 2-1). Another method used for subcritical areas employs the superposition of two critical width profiles.

Similar limitations as for the NCB graphical method are encountered for the profile function method. Although it is fairly easy to use, several arbitrary constants and coefficients characterizing the entire

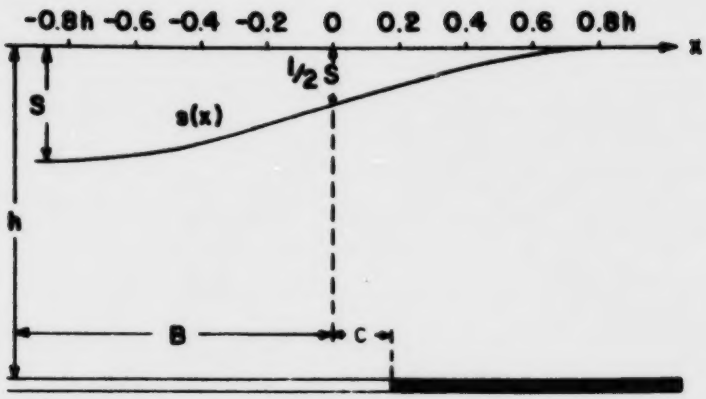

Figure 2-2. A General Subsidence Profile Function (From Brauner, 1973).

TABLE 2-1. Profile Functions (After Brauner, 1973)

	Profile Function	Area of Application
1) Hyperbolic	$s(x)=0.5\ S\ (1-\tanh(bx/B)$	U.K.
2) Error	$s(x)=0.5\ S\ (1-(2/\sqrt{\pi}\int_0^{(\pi x/B)} e^{-u^2} du$	Upper Silesia Poland
3) Exponential	$s(x)=S\ \exp(-(1/2)[(x+b)/B]^2$	Hungary
4) Donets Coal Field	$s(x)=0.5\ S(1-(x/B)-(1/\pi)\sin\pi(x/B))$	USSR
5) Trigonometric	$s(x)=S\ \sin^2[(\pi/4)(x/B-1)]$	
6) Donet's Coal Field	$S(x) = S\left\{n^2(1-x_o+\frac{\sin 2\pi x_o}{2\pi}+\frac{(1-n^2)}{4}(1+\cos\pi x_o)^2\right\}$	Sub Critical Areas ($n>1/3$)
7)	$S(x) = \frac{1}{4} S (1+\cos\pi x_o)^2$	Subcritical Areas ($n<1/3$)

Where:

$n = .9\ w/h$

$x_o = x/L$

$L = \frac{1}{2}w + h\tan a$

w = width

h = depth

a = limit angle

b = arbitrary constant

overburden rock mass must be determined from the field data. Each specific function pertains only to a given district from which field measurements have been obtained. Thus, the profile function method is not applicable to the Cape Breton situation until seabed subsidence data are available.

2.4 INFLUENCE FUNCTIONS

The influence function method is based on the postulate that the effects of extraction on the subsidence of a point on the surface can be superimposed. An excavation can be regarded as being composed of an infinite number of infinitesimal elements. The mathematical relationship that establishes the displacement of a surface point due to that infinitesimial extraction volume is known as the influence function. The subsidence of a surface point can be obtained by integrating the influences of all the infinitesimal elements of excavation. The method is very versatile, suffering none of the geometrical restriction of the profile function (U.S. Bureau of Mines, 1983).

The fundamental concept of the influence function is illustrated in Figure 2-3, showing that the total subsidence curve is the summation of all the individual elemental subsidence curves. The model also assumes that the subsidence of the surface is proportional to the volume (or area for a constant seam thickness) of the material extracted from the seam. The contribution of each excavated element on the subsidence of the overlying point is the product of its volume dV, (or area, dA), and the magnitude of the influence that dA exerts on that surface point. This influence on the surface point, p, is a function of the radial distance from dA and can be expressed mathematically by $p = p(r)$. This influence function for many cases has a maximum value at r=0 and diminishes as r increases. The subsidence of a point can therefore be expressed as the integral of the influence function over the entire area of influence A of extracted material.

$$S(r) = \int_A p(r)\, dA \qquad (2\text{-}2)$$

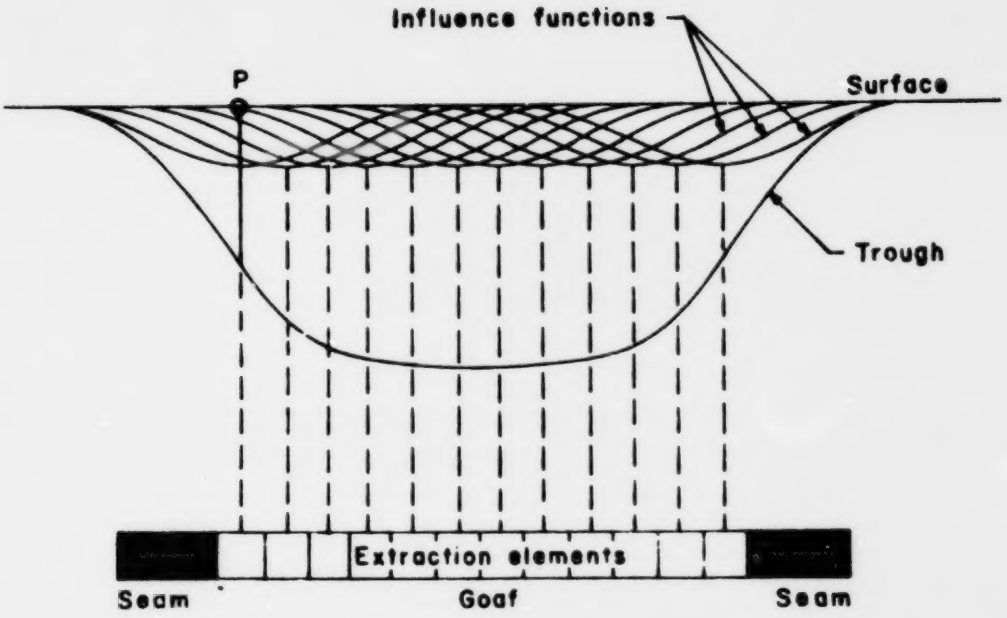

Figure 2-3. Superposition of Elemental Subsidence Curves to Yield the Total Subsidence Trough (After Brauner, 1973).

When polar coordinates are used, $dA = rdrd\theta$ and Equation 2-2 becomes

$$S(r) = \int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2} p(r)\, rd\theta dr \qquad (2\text{-}3)$$

From Equation 2-3 it can be seen that the subsidence at a point is the volume under the curve p(r) contained in the area defined by r_1, r_2, θ_1 and θ_2.

Hood et al (1981) noted that the fundamental assumption regarding each elemental extracted area as having the same subsidence effect at the surface is equivalent to saying that the overburden behaves in a homogenous, isotropic manner. Also, they noted that integration of all the elemental extracted areas across the mined area gives the profile function and, thus, a profile function can be derived from an influence function in principle. Further discussion on the relationship between these functions is given by Munson and Eichfeld (1980) and Hood et al (1981).

Many influence functions have been developed for the central European coal fields and, in practice, about half a dozen influence functions have been used with various degrees of success (U.S. Bureau of Mines, 1983). A list of selected functions is given by Brauner (1973) and is shown in Table 2-2. Examination of the functions listed shows that they are of the form:

$$P(r) = k_1\, S\, f(R, r, k_2) \qquad (2\text{-}4)$$

where S = subsidence at the panel centre
R = the critical radius
r = the radial distance from the point under consideration
k_1, k_2 = constants

The behavior of these functions has been investigated by Hood, Ewy and Riddle (1981). They concluded that more than one function may be employed in order to obtain an accurate predictive tool for the complete

Table 2-2. Influence Functions (After Brauner, 1973)

1) $p(r)=[(S)(R^3\tan^3\beta))]/\pi[[\sin\beta\cos\beta+(\pi/2)-\beta)][r(r^2+R^2\tan^2\beta)]^2]$,

where: β=Break angle

2) $p(r)=3(S)[1-(r/R)^2]^2/\pi R^2$

3) $p(r)=2(S)\exp[-4(r/R)^2]/(\pi^{1.5}Rr)$

4) $p(r)=0.216(S)\exp[-4(r/R)^6]/Rr$

5) $p(r)=(S)/\exp[-(r/R)^2]/R^2$

6) $p(r)=4.6(S)/\exp[-4.6(r/R)^2]/\pi R^2$

7) $p(r)=2(S)\exp(-2(r/R)^2]/R^2$

8) $p(r)=7(S)/\exp(-6.65\times r/R)/R^2$

9) $p(r)=C(R_1(r)^{-3}-R_2(r)^{-3})$

trough, and that in some cases a shift of location of the predicted inflection point may be required.

Equation 1 (Table 2-2) is obtained from the solution of the subsidence problem within the framework of linear elasticity in which the computation of surface displacements can be regarded as a boundary value problem. In this regard it actually forms an analytical rather than empirical method. Equation 9 (Table 2-2) will be discussed later.

The influence function method has the distinct advantage that it is capable of handling uniform as well as non-uniform (three-dimensional) extraction patterns and can model both longwall and room and pillar mining systems. Its theory implies the acceptance of the law of superposition and there is no restriction on the limit angle. The assumption that each element has the same subsidence effect is equivalent to saying that the overburden behaves in a homogeneous, isotropic manner (Hood et al, 1981). Its accuracy of prediction also depends on the parameters S and R given in Equation 2-4, subsidence at the panel centre and critical radius, respectively. These equation parameters are interdependent; if one of them is changed without the other then errors in the subsidence profile may occur (Hood et al, 1981). The main difficulty associated with empirical influence functions is the lack of uniqueness in the relation between subsidence and its cause. A secondary practical difficulty is the need for adjustments near the excavation edges, usually in the form of compensation zones (Brauner, 1973).

Other methods which are based on the theory of influence functions are:

1. Zone Area Method
2. Complementary Influence Functions

These techniques are refined developments of the influence functions and are well suited for computer analysis.

2.4.1 Zone Area Method

The zone area method is well suited for computer application and is relatively widely used. It has the advantage that multiple panels and three-dimensional workings can be modeled.

The estimation of subsidence is carried out by constructing a series of circular zones or concentric rings around the surface point and determining the proportion of each zone or ring which has been extracted. Various factors (or functions) are applied to these proportional areas which are then totalled to give the subsidence value (Marr, 1975). Using this procedure, the subsidence of a surface point is given by the expression:

$$S = aA + bB + cC + ... \qquad (2\text{-}5)$$

where S = subsidence as a proportion of the seam thickness
a,b,c, etc. = zone factors
A,B, etc. = proportional areas extracted of rings A,B,etc.

For a system of n zones and n known subsidences, a set of n equations can be assembled to solve for the n unknown zone factors.

For manual calculations, Marr (1975) recommended a system of seven zones, each zone having a convenient width of 0.1 of the depth of the working. Figure 2-4 shows the ring construction for subcritical, critical and supercritical cases.

When solving for the subsidence directly over the edge (i.e. ribside) of a critical or super-critical excavation, one half of the influence area would be excavated, implying one half of maximum subsidence at that point and inflection of the subsidence profile at that point. Unfortunately, in many cases the inflection point is shifted over the excavated area. The zone area method can be modified to allow for this deviation by deducting a "compensation" zone from the panel which is treated as if it were not mined. Another method of modifying the zone area method when the location of the half maximum subsidence surface point does not coincide with the edge of the workings (ribside), is to use a non-linear relationship between zone factors and extracted areas. Marr (1975) postulated using a power series in the form of

$$S = aA^n + bB^n + cC^n + ... gG^n \qquad (2\text{-}6)$$

where n = a constant.

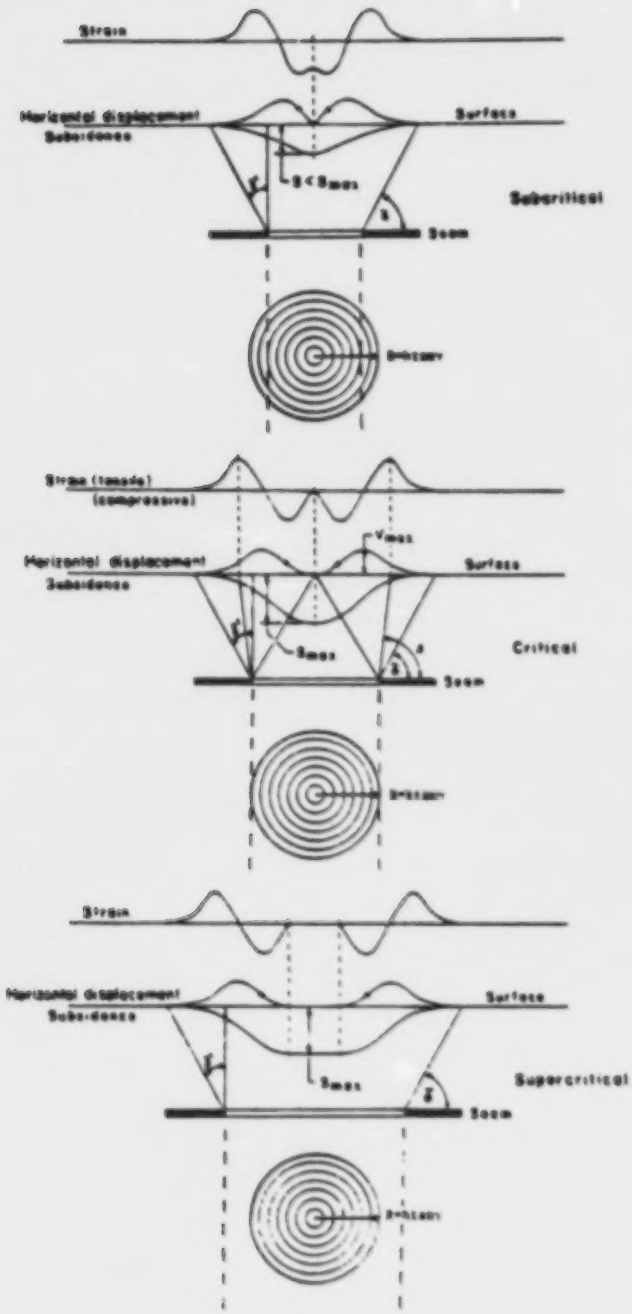

Figure 2-4. Ring Construction for the Zone Area Method for Subcritical, Critical and Supercritical Cases (After Karmis et al., 1981).

In applying the above equation, the zone factors and the influence constant must be determined from the field. For the case of complete extraction, the subsidence factor is given by

$$S. = a1^n + b1^n + C1^n + ... g1^n \qquad (2\text{-}7)$$

In addition, above the ribside each zone has half of the area under consideration extracted and the subsidence factor becomes

$$m = a(1/2)^n + b(1/2)^n + ... + g(1/2)^n \qquad (2\text{-}8)$$

where $S.$ = maximum subsidence
m = subsidence over the ribside

By solving equations (2-7) and (2-8), n can be determined as

$$n = \frac{\ln (S.) - \ln(m)}{\ln(2)} \qquad (2\text{-}9)$$

In cases of subcritical extractions, the above equation must be modified in the following form:

$$n = \frac{\ln(S.) - \ln(m)}{\ln(2) - \ln(Ae)} \qquad (2\text{-}10)$$

where Ae = the proportional extracted area

The values of zone factors (a,b,c... etc.) and the constant n for British coalfields are well established (Marr, 1975) and are given in Table 2-3, along with the constants appropriate for the eastern and western U.S.A.

Observations in the British coalfields have shown that the simple addition of two overlapping subsidence profiles produced by adjacent working faces is greater than the observed values over the centres of coal pillars, unless the pillar has a width/depth ratio of 0.5 or more. Marr (1975) suggested that adjustment to the basic zone area system to allow for the

Table 2-3. Zone Factors for UK and USA Coalfields

Factors (see Equation 2-6)	UK[1]	Eastern[2] USA	Western[3] USA
a (inner zone)	.056	.050	.030
b	.199	.144	.112
c	.259	.159	.134
d	.219	.188	.219
e	.109	.100	.092
f	.045	.034	.031
g (outer zone)	.013	.005	.006
n	2.296	3.61	2.678
Angle of Draw	35°	28°	21°

1. From Marr (1975)
2. From Karmis et al (1981)
3. From C.L. Stewart (personal communication)

effect produced by the coal pillar should be applied. The area of influence of pillars was divided into five concentric rings with zone widths of 0.1 of the depth of working and a reduction profile suggested using the following expression:

$$R = vV + wW + xX + yY + zZ \qquad (2\text{-}11)$$

where R = pillar reduction value
V.....Z = proportional pillar area
v.....z = pillar zone factors

The estimation, as given in Equation (2-11), of a reduction value is related to the size and shape of the coal pillar and can be carried out using the basic zone area method. An additional correction value has to be applied to R based on observed subsidence profiles. Methods and constants for applying these correction factors are described by Marr (1975) for British coalfields.

For seams inclined up to approximately 20°, observations have shown that the maximum subsidence occurs at a position above the centre of the face perpendicular to the seam gradient (Marr 1975). Observations indicate that the shape of the zone areas remains circular and, thus, the zone area method is applicable. Very little information is available for coal mines with inclinations greater than 20° and it is recommended that the zone metho not d be used for these steeper gradients.

2.4.2 Complementary Influence Functions

The computation difficulties experienced with influence functions in determining the displacements above the ribside were examined by Sutherland and Munson (1984). They proposed the concept of complementary influence functions to describe the response of mined and unmined zones acting together to produce the observed subsidence. Each influence function is defined by the response of two types of unit element: an "unmined" element for coal left by the mining process, and a "mined" element for the open volume created by the removal of the coal. The elements are shown in Figure 2-5.

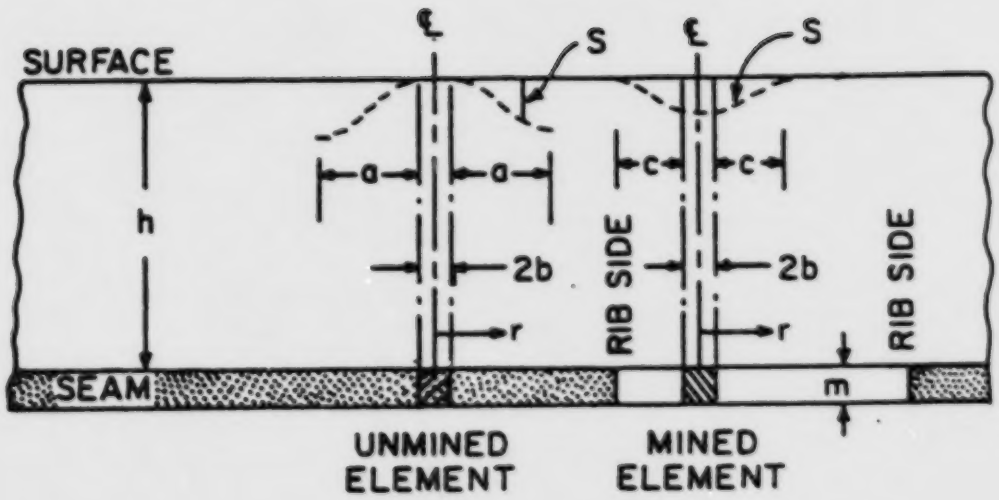

Figure 2-5. Schematic Figure Showing the Use of Complementary Influence Function Elements (After Sutherland and Munson, 1984).

When these elements are integrated over the entire seam (mined and unmined) a subsidence prediction can be obtained. The subsidence is the sum of the influence of the mined and unmined response.

The elemental response of the unmined element is based on the elastic response of a plate supported by the element (Timoshenko and Woinowsky-Kreiger, 1959). This approach is clearly an analytical method but has been included here for convenience due to its similarity to the influence function method. The response is given by the function

$$S/m = P_S \begin{cases} 0 & 0<r<b \\ [I]\,[C]\,[r_s^2\,(1-2\,\ln r_s^2)] & b<r<a+b \\ 1 & a+b<r \end{cases} \qquad (2\text{-}12)$$

where $r_s = r/a - b/a$

S = vertical subsidence around unmined element

m = mined height

P_S = proportion of maximum subsidence attributed to unmined element

r = radial coordinate from centre of element

b = half width of element

a = radial extent of influence

$[I]$ = effect of moment of inertia of overlying strata

$[C]$ = effect of crushing of unmined material on element response

Numerical and experimental models of the response of the overlying strata (Sutherland and Schuler, 1981) have shown that the failure zone above a longwall panel produces a thinning of the remaining elastic beam as one moves away from the rib. The elastic solution can be modified to take into account this thinning by using the moment of inertia function:

$$[I] = [t./t(r_s)]3 \qquad (2\text{-}13)$$

where $t(r_s)$ = thickness of plate as a function of radius

$t.$ = centre thickness of plate

An error function, based on the probability of the migration of voids through the caved overburden is used to analyse the response of the mined element:

$$S/m = P_v \begin{cases} 1 & 0 < r \leq b \\ \text{erfc}\ (2r_v) = 1 - 2\pi \int_0^{2r_v} e^{-l^2}\, dl & b < r < c+b \\ 0 & c+b < r \end{cases} \qquad (2\text{-}14)$$

where
- S = vertical displacement due to mined element
- r_v = r/c-r/b
- P_v = proportion of maximum subsidence due to mined element
- c = radial extent of influence outside the element
- l = integration variable
- erfc = complementary error function

According to Equations (2-12) and (2-14), the parameter set that governs subsidence is a, c, P_s and P_v. In terms of a longwall panel, a and c can be interpreted as the half range of the profile function and the draw angle, and P_s and P_v are related to the centreline displacement and the displacement above the ribside, respectively. The application of complementary functions to three-dimensional panel geometry shows the subsidence profiles are in quantitative agreement with field measurements (Sutherland and Munson, 1984).

2.5 INCREMENTAL DESIGN

Incremental design is the process of extrapolation from the known to the new, or of predicting the conditions that result from a change in the present operations. The basic assumption is that prediction of coal mine subsidence can be made based on previous experience.

For flat-lying ore bodies the following equation can be used for determining the maximum vertical subsidence that will occur over a new mining panel for identical geological conditions (Coates and Gyenge, 1973).

$$S^b/S^a = [(L_b/L_a)\ (Y_b/Y_a)\ (B_b/B_a)\ (A_b/A_a)]^{1.5}\ (Z_a/Z_b)^3 \times (H_b/H_a) \qquad (2\text{-}15)$$

where
- a = superscript for previous case
- b = superscript for future case
- L = length of the panel

Y = span of the panel
Z = depth of the workings
H = thickness of the workings of a 100% extraction or maximum possible closure for less than 100% extraction.
B = empirical factor depending on the L/Z ratio
A = empirical factor depending on the Y/Z ratio
(e.g. for NCB data, B = 2.8 - L/Z and A = 2.8 - Y/Z)

For the case of L/Z = 1.4, Equation (2-14) reduces to:

$$S^b/S^a = [\ (Y_b/Y_a)\ (Z_a/Z_b)\ (A_b/A_a)\ ]^{1.5} \times (H_a/H_b) \qquad (2\text{-}16)$$

Over massive coal bodies, where mining eventually leads to cover caving, the subsidence preceding caving can be treated essentially the same as for the flat-lying seams.

The incremental design requires previous data on subsidence and equations so derived assume previous and future cases are essentially the same. A large collection of data has to be compiled over a long period of time. It appears that this method has never actually been used for subsidence prediction.

2.6 STOCHASTIC METHODS

A number of stochastic models of the subsidence process using probabilistic methods were developed in the 1950's and 1960's (for example, Litwiniszyn, 1957; Sweet and Bogdanoff, 1965). Since these methods do not specifically model subsidence mechanisms, they are considered to be empirical.

The conceptual model of a stochastic medium was originally developed by Litwiniszyn (1957) as a means of describing subsidence over underground mine excavations. The objective was to determine a probability that the value of subsidence is less than or equal to some constant. The model consisted of a layered sequence of small cubical cells, each containing a ball. If a ball is removed from a lower cell, it is replaced with equal probability by a ball from either of two cells directly above. Clearly, the downward movement of a ball is equivalent to an upward movement of an empty cell or void. The propagation of void from initial mine cavity to the

surface is similar to a random walk and probability distributions and equations can be developed.

The stochastic model describes the behavior of a collection of many discrete members or particles and may well be applicable to soils and perhaps specialized cases such as block caving. However, its application to layered competent strata does not appear warranted. This may be the reason that virtually no work in this area has been done in the past ten years.

2.7 DISCUSSION

This section has briefly reviewed a number of empirical subsidence prediction methods including the National Coal Board graphical method, profile functions, influence functions, incremental design and stochastic methods. Some of these methods are based on a substantial amount of field observations and have been successfully used in coal mine design.

All of the empirical methods have the major limitation that no field data are currently available from the Sydney coalfield and, furthermore, it is unlikely that much data will be available in future years. However, it may be possible to 'calibrate' the profile and influence function methods using data from similar geologic situations in the eastern U.S.A. In this regard, it is felt that the latter methods, particularly the zone method, may have some applicability to the Sydney coalfield.

A promising extension to the empirical techniques is the 'hybrid' method which combines the usual empirical influence function approach with rigorous statistical curve fitting; it can also employ influence functions which are based on engineering and physical models such as beam bending, beam on an elastic foundation, or similar. Regression analysis (e.g. least squares fitting) of the predicted and actual measured subsidence data can be made to derive the function parameters and the validity of the model can be evaluated using correlation coefficients such as those given by Draper and Smith (1981). The hybrid method provides a powerful tool for duplicating past subsidence and predicting future subsidence when a large base of field observations exists.

3 ANALYTICAL SUBSIDENCE PREDICTION METHODS

3.1 INTRODUCTION

Historically, the early analytical theories, which were largely descriptive, were based on fracture of the overburden and rules for locating the boundaries of the broken material. The introduction of mathematical concepts soon forced theoretical developments away from the treatment of fracture, and into analytical theories of elastic continuum displacements. The continuum theories evolved from the early analysis of Hackett (1959) of a displacement discontinuity produced by excavation of a thin seam in an elastic medium. A series of extensions by Berry (1960), Berry and Sales (1961, 1962) and Salamon (1963) have treated the semi-infinite medium for two dimensional isotropic, transversely isotropic and three-dimensional transversely isotropic elastic behavior. The three-dimensional theory was later modified by Avasthi and Harloff (1982) to bring the subsidence profiles into agreement with the subsidence data observed by the U.K. National Coal Board (1975), and extend its application to multiple cavities and steeply dipping seams.

However, the principal difficulty, as described by Munson and Benzley (1980), was the need to obtain compatible elastic moduli in order to obtain agreement with the field values. The void-volume distribution functions were used in order to re-examine the natural phenomena occuring during subsidence.

Other attempts to overcome these deficiencies through more elaborate constitutive models of stratified ground, and non-linear and plastic material behavior were made possible by numerical techniques. The finite element method is the most commonly used method in the application of mechanics to the subsidence problem. The greatest strength of the method is the relative ease in treating non-homogeneous and non-linear media. Extremely complicated mine geometry can also be accomodated. Representative examples of using this method include Ishijima and Isobe (1973), Dahl and Choi (1973), Langland and Fletcher (1976), Ratigan and Goodman (1981). An alternative to this method is to use the finite-difference method. Codes utilizing an explicit finite difference formulation are more amenable to large-strain, non-linear modeling such as occur in subsidence (Trent, 1981). Trent (1981) concluded that the costs of explicit finite-difference calculations are significantly lower in three-dimensional models than similar finite element programs.

Other elastic approaches which employ numerical techniques concentrate on "displacement discontinuity" (Crouch, 1976), "boundary integral equation" (e.g. Lachat and Watson, 1976) and "boundary element" (e.g. Tsur-Lavie and Denekamp, 1978) type of solutions. These methods may be termed as "surface element" techniques, because they require only the boundaries (i.e. the surfaces) of a particular region be discretized. The finite difference and finite element methods require the discretization of the entire region (including the boundaries) and therefore are referred to as "volume element" techniques. The system of equations resulting from the "surface element" technique is smaller than those from the "volume element" technique (Lachat and Watson, 1976).

3.2 CLOSED-FORM ELASTIC SOLUTION

Although much research has been concentrated on complex situations in which the deformation of the ground is due to several different mechanisms operating in different regions, it is reasonable to assume that the ground can still be considered to behave according to one or other of the constitutive equations of continuum mechanics over most of its extent (Berry, 1978). The continuum theories discussed in this section evolved from the analysis of a displacement discontinuity produced by a slit in an infinite half-space elastic media. Berry (1960) analyzed elastic ground movement for three kinds of underground excavation development: (1) nonclosure, (2) partial closure, and (3) complete closure. He assumed that the thickness of the extracted seam, t, is small compared with the depth, h, and width, 2a, of the excavation. In his treatment the lower boundary of the seam is given the same coordinate as the nearest point on the upper boundary. Then the convergence of opposing points in roof and floor can be treated as a discontinuity in displacement at a single point. Berry (1960) treated excavations as displacement discontinuities by using a complex potential method for partial closure. The results were then extended to nonclosure and complete closure. The two-dimensional version was solved exactly for the limiting case of complete closure, and approximately for less than complete closure.

Berry and Sales (1961) extended the closed-form solution to the case of transversely isotropic ground in two dimensions. Based on observations with

existing data, Berry (1963, 1964 and 1978) subsequently rejected the isotropic solutions for British conditions. The resulting expression for complete closure over -a x a at depth, h, in two-dimensional, anisotropic plane strain problems with great depth (h a) is given by the following equation:

$$s(x) = \frac{-2^{1/2} \cdot .2 \text{ at } h^3 (k_1 + k_2)^{1/2}}{\pi (k_1^2 x^4 + 2k_2 h^2 x^2 + h^4)} \qquad (3\text{-}1)$$

where

s(x) = vertical subsidence at the surface as a function of the horizontal distance, x

2a = panel width

h = seam depth

t = seam thickness

$k_1 = (1-V_1^2)^{1/2} (E1/E_2-V_2^2)^{-1/2}$

$k_2 = [E_1/2G-V_2(1 + V_1)] (E1/E2-V_2^2)^{-1}$

E_1, E_2 = Young's Moduli for transversely isotropic media (parallel and perpendicular to bedding)

V_1, V_2 = Poisson ratio (parallel and perpendicular to bedding)

G = shear modulus

The transversely isotropic model is of considerable practical significance because it shows that the behaviour of a stratified rock mass can be approximated by determining equivalent elastic constants (Salamon, 1968, Singh and Kumar, 1973 and Berry, 1978). It is possible to calculate these equivalent elastic constants from field measurements (e.g. shear and compression wave velocities) or from laboratory measurements of the rock properties. (In practice, however, it is often difficult to obtain representative values).

Both the isotropic and the transversely isotropic elastic models were studied by Daemen and Hood (1981). Attempts were made to match observed subsidence with predicted values using three independent variables: the elastic constants k_1 and k_2, and the elastic closure. Their results did not produce satisfactory curve fitting and they concluded that a consistent elastic approach to subsidence calculations for American coal mines will require the use of three-dimensional solutions.

The complete closure of a rectangular panel excavated to a uniform thickness is considered as a constant displacement discontinuity in a three-dimensional, transversely isotropic and homogeneous half-space. The results of the three-dimensional theory developed by Berry and Sales (1962) are presented below. The coordinate system used in this development is shown in Figure 3-1.

The surface vertical displacement equation is given by ($k_1 > k_2$, see Equation 3-1):

$$s(x,y)/t = \frac{-1}{4\pi}\left[\frac{1}{2}\left(\frac{k_1 + k_2}{k_1 - k_2}\right)^{1/2}\sum_{n=1}^{4}\frac{\ln B_n + C_n X_n^2 + k_2 X_n^4}{B_n - C_n X_n^2 + k_1 X_n^4} + \sum_{n=1}^{4}\frac{\tan^{-1} D_n X_n^2}{B_n - k_1 X_n^4}\right] \quad (3\text{-}2)$$

where:

$$A_n = Y_n^2 + k_2^2/k_1^2$$
$$B_n = [A_n^2 + (k_1^2 - k_2^2)/k_1^4]^{1/2}$$
$$C_n = 2^{1/2}\,(k_1 B_n - k_2 A_n + k_2^2/k_1^2 + 1)^{1/2}$$
$$D_n = (4k_1 B_n - C_n^2)^{1/2}$$

$$X_1^2 = (a-x)\,(b-y)/h^2 \qquad X_2^2 = (a+x)\,(b-y)/h^2$$
$$X_3^2 = (a+x)\,(b+y)/h^2 \qquad X_4^2 = (a-x)\,(b+y)/h^2$$

$$Y_1^2 = [(a-x)^2 + (b-y)^2]/h^2 \qquad Y_2^2 = [(a+x)^2 + (b-y)^2]/h^2$$
$$Y_3^2 = [(a+x)^2 + (b+y)^2]/h^2 \qquad Y_4^2 = [(a-x)^2 + (b+y)^2]/h^2$$

The three-dimensional theory was studied by Daemen and Hood (1981) and Avasthi and Harloff (1982). Both authors indicated that a reasonable fit between observed field data and predictive values is possible. Following a procedure of extensive statistical fitting of NCB data, Avasthi and Harloff (1982) put forward a modified version of Berry and Sales' work which is referred to here as the "modified three dimensional model". The modified model has the following capabilities:

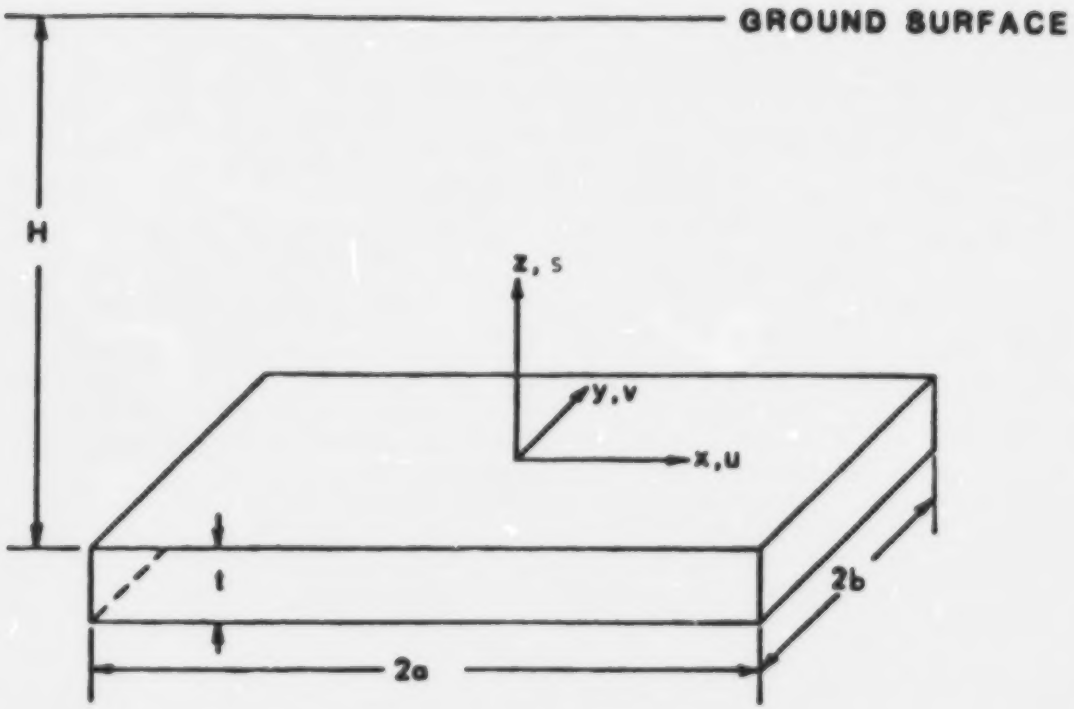

Figure 3-1. Schematic Figure of the Layout and Coordinate System Used in the Closed Form Solution. (After Avasthi and Harloff, 1982)

i) The empirical subsidence problems from the NCB data can be adequately modeled with theory.

ii) Seam thickness is no longer an adjustable parameter.

iii) Extension to multiple cavities is possible.

iv) The model can be applied to steeply dipping seams.

Two empirical transformations $x = 2x$ and $y = 2y$ were made to bring the theoretical profiles into a narrower extent as given by the NCB data. The horizontal displacement equations were rearranged using complex algebra theory and some signs in the original formulation were changed to satisfy symmetry. A detailed discussion can be found in the original report (Avasthi and Harloff, 1982). The following equations are obtained:

$$u(x,y)/t = \frac{k_1}{4\pi N} \left[\tan^{-1} K_{121} - \tan^{-1} K_{221} + \tan^{-1} K_{322} - \tan^{-1} K_{422}\right] \tag{3-3}$$

$$v(x,y)/t = \frac{k_1}{4\pi N} \left[\tan^{-1} K_{111} - \tan^{-1} K_{211} + \tan^{-1} K_{312} - \tan^{-1} K_{412}\right]$$

where:

$$\tan^{-1} K_{ijk} = \tan^{-1} \frac{2\left\{-(B_n+A_n)^{1/2} + (2/H)(-1)^{k-1} R_{jk}\right\}\left[B_n-A_n\right]^{1/2}}{\left\{-(B_n-A_n)^{1/2} + 2/H(-1)^{k-1} R_{jk}\right\}^2 - (B_n-A_n)}$$

$$R_{11} = a - 2x \qquad R_{12} = a + 2x$$

$$R_{21} = b - 2y \qquad R_{22} = a + 2y \qquad N = [(K_2 - K_1)/2]^{1/2}$$

where A_n, B_n were previously defined. The resulting displacement equations can be applied to determine surface strain profiles which are the first derivatives of the displacement curves. It must be stressed that this is effectively an empirical formulation which has been formed by constraining the analytical results to fit the NCB curves. It is included in the analytical section for convenience.

Empirical adjustments for half-width/depth 0.3 are required to account for incomplete closure. This appears to be the reason for the fact that observed subsidence is smaller than predicted. The factor is given by:

$$\text{Factor} = 0.62 + 5.01\,(a/H)^{2.14} \qquad (3\text{-}4)$$

for $(a/H) \leq 0.3$

where: a = half width of panel
H = depth of seam

The modified model suggests that the values of k_1 and k_2 are 3.965 and 1.79, respectively, in order to provide satisfactory agreement with the NCB subsidence data. These values become unity for homogeneous isotropic ground. In the case of multi-layered media, solutions can be obtained by using an averaging process for elastic constants. A more realistic approach is to use field subsidence data for determining the elastic constants k_1 and k_2. This requires a curve fitting procedure.

Superposition is used in the modified model to obtain subsidence profiles for multiple seams. Subsidence at any point on the surface is given by the summation of the individual subsidence profiles.

The model was also applied to a steeply dipping coal seam (Avasthi and Harloff, 1982). The depth of seam has to be adjusted to an equivalent case of a horizontal seam and the profile projected appropriately onto the surface, as illustrated in Figure 3-2. The subsidence profile obtained for the horizontal case is then converted back to that of the steeply dipping case, for example, Abel and Gentry (1978). A plot of limit angles versus angle of dip of seam, e.g. those given in the NCB handbook, is required for projecting the equivalent predicted profile onto the original horizontal ground surface. It was also concluded that reasonable strain results could be obtained using the theory.

The study of hydraulic fractures has led to a number of other elastic formulations for vertical displacement, e.g. Pollard and Holzhausen (1979), Sun (1969), Evans (1983) and others. The term 'dislocation' is generally used which bears the same meaning as "displacement" in Berry's treatment. Analytical expressions for displacemnt are obtained based on the theory of elasticity and the shape of the underground cavity (crack) is usually elliptical or circular. The developments are similar to that described in this section. Sun (1969) analyzed the problem of a circular disc in a

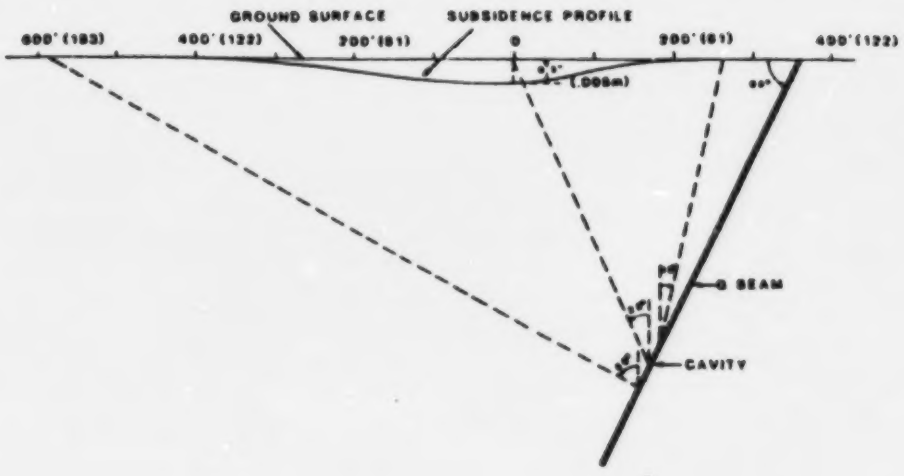

Figure 3-2. Subsidence Analyses over a Steeply Dipping Seam. (After Avasthi and Harloff, 1982).

semi-infinite half-space with a pressure on the disc or crack surface. The semi-infinite media was assumed to be homogeneous, isotropic and linear elastic.

Several other analytic models were studied by Evans (1983) and he concluded that observed surface movements are consistent with the prediction of elastic continuum models of hydraulic fractures.

The general approach for analyzing underground fluid-filled fractures and underground coal gasification is also similar and closely related.

The closed-form elastic solutions for subsidence prediction described in Section 3.1 were originally rather restricted in the complexity of mining situations they could model. However, extensions to the theory now allow dipping and multiple seams as well as three-dimensional (rectangular) panels. A distinct advantage of this method is that, once it is programmed for a computer, it is easy and cheap to use, requiring relatively simple data input (see the Appendix for example).

There are also some distinct limitations to this method. First, since it is based on elastic theory, it cannot model more complex subsidence mechanisms such as roof failure and bed separation. Second, the mathematical formulation assumes that the seam is completely closed after mining and this may not be totally valid due to bulking of caved material or incomplete closure or other factors. Another limitation is that the entire rock mass is assumed to be homogeneous with elastic constants K_1 and K_2 (see Section 3.1).
Although methods have been developed to calculate equivalent elastic moduli to represent a sequence of different geologic strata, it is often difficult to obtain sufficient rock property data to make this calculation. The method does not allow for irregular (i.e. non-stratified) inhomogeneities.

3.3 NUMERICAL METHODS

Numerical models provide the most general tool for the quantitative analysis of subsidence and related strata-mechanics problems. They are not subject to the restrictive assumptions required for closed-form analytical solutions and offer considerable flexibility in dealing with complex stratigraphy and mine geometry. In addition, a number of non-linear phenomena can also be modeled by modifying the methods appropriately.

Numerical techniques for solving solid mechanics can be generalized into two categories: (1) the finite difference and finite element methods which require that the entire volume (area) under consideration be divided into a mesh and (2) the boundary integral methods, in which only the bounding surface of the region under consideration is subdivided. All of these methods have been used in subsidence research and will be discussed.

3.3.1 The Finite Difference Method

The finite difference method (FDM) approximates continuous partial differential equations with a set of discrete equations in space (and time). The approximations to derivatives are made by finite differences which are similar to the derivatives they approximate. Because this theory is easier to understand, the programming of computer codes is simpler than for the finite element or boundary integral methods.

To apply the finite difference method, a grid is established for the region of interest, over which the relevant equations are approximated. In general the grids are rectangular and regular, although less commonly used methods such as the integrated finite-difference method utilize an arbitrary grid.

With this method an algebraic equation results for each nodal point in the mesh yielding a set of simultaneous algebraic equations. This matrix equation is solved by applying either direct or iterative solution procedures.

The finite difference method is not generally used in applications involving the mechanics of solids because the rectangular grid cannot readily approximate non-rectangular shapes or handle tensorial parameters with cross-product terms. However, these difficulties can be overcome by specialized treatments and some powerful computer programs have been developed which are applicable to subsidence prediction. Furthermore, the explicit formulation of the finite-difference equations allows complicated, non-linear, large-strain material behavior to be modeled in a relatively straight-forward procedure (Trent, 1981). This allows some mechanisms, such as caving of the roof, to be included in subsidence analyses.

Excellent reviews of the finite difference method can be found in Forsythe and Wasow (1960) and Faust and Mercer (1980).

3.3.2 The Finite Element Method

The finite element method (FEM) is the major numerical technique presently used in the analysis of solid mechanics problems and much research effort has been directed toward extending its capabilities to specialized and non-linear applications. A wide variety of finite-element computer programs are available for subsidence prediction. Finite-element applications to subsidence problems have been performed by Dahl and Choi (1972), Ishijima and Isobe (1973), Dahl and Choi (1975), Munson and Benzeley (1980) and Ratigan and Goodman (1981), just to name a few.

In this method, the region of interest is modeled as a finite number of idealized elements, in either two or three dimensions. The elements may be of a variety of shapes and may even include curved sides. This flexibility of elements enables consideration of regions with very complex geometry. The governing partial differential equations are converted to an integral representation using one of several methods such as the principle of virtual work, minimization of potential energy, or a variational method. Unlike the finite-difference method, the mathematical formulation is not intuitive and implementation in a computer program is generally more difficult. The formulation results in a set of simultaneous, algebraic equations with the nodal displacements as the unknowns. The matrix equations are solved using direct or iterative procedures and then the stresses and strains are calculated from the displacements.

As for the finite-difference method, this method has been extended to include a number of non-linear capabilities such as large-strains, post-failure behavior and slip along joints or faults.

Extensive literature is available describing the finite element method, for example, Zienkiewicz (1977), Cook (1981), Bathe (1982), Owen and Hinton (1980).

A comparative study was conducted by Trent (1981) using a linear-elastic, finite-element program and a finite-difference computer program. As would be expected, the results obtained were nearly identical in terms of stresses and displacements.

The finite element and the finite difference formulations have been employed for subsidence prediction. They provide a general approach for describing the displacement and stress field induced throughout the rock mass. The most attractive feature of these approaches is the facility with which inhomogeneity can be treated as well as complex mining configurations, different types of discontinuities and complicated surface topography. Models that have been used for subsidence studies include various types of linear elasticity, non-linear and elasto-plastic material formulations to simulate failure zones and viscoelastic equations to account for time-dependent subsidence.

One of the principal disadvantages of the finite element or finite difference method is that it requires the introducion of a large number of variables. It is very difficult to obtain all the necessary input data to the required degree of reliability. Due to the large volume of data involved it may be possible to inadvertently correct an erroneous assumption by introducing a compensating one, for example, to magnify displacement by using unrealistically soft rock properties. Although these numerical analyses methods can incorporate non-linear behavior such as roof caving, the procedures for doing so are far from standard and should be regarded with some caution. A major disadvantage to both the FEM and FDM is that all the major non-linear programs are very time-consuming and cumbersome to use and require considerable experience to ensure that valid results are being obtained.

3.3.3 Boundary Integral Methods

Numerical analysis methods that require only the boundaries of a particular region to be discretized are sometimes termed "surface element" techniques. Other formulations which fall in the same category include the "face element method", "boundary element method" and "displacement discontinuity" or "influence function method". The fundamental distinction among these methods is the way in which a system of algebraic equations is obtained by employing a different elementary solution. For example, the "displacement discontinuity method" given by Crouch (1976), is a solution to the problem of a constant displacement discontinuity, or dislocation, over a finite line segment in an elastic body (see Section 3.2). The "boundary

element method" given by Tsur-Lavie and Denekamp (1978) is a solution for a single step-like displacement consisting of a discontinuous and uniform displacement at the boundary of an infinite half plane. The close connection among these types of solutions make it clear that the distinction is becoming somewhat artificial.

The early development came from the introduction of the "face element principle" as a basis for treating the irregular layout of tabular excavations (Salamon, 1963, 1964 and 1964). The idea is similar to the influence function concept, except that the displacement of a point is based on elastic analysis. This concept makes use of the linearity of the equations of elasticity and expresses all displacement and stress components induced by mining as a summation (integral) of elementary displacements and stresses which are due to vertical and horizontal components at an arbitrary point in the plane of the deposit. Salamon (1963) expressed the vertical and horizontal displacements as a function of the convergence of the face elements, problem geometry and certain influence functions. In a number of instances, such as the homogeneous, isotropic medium, the influence functions have been determined (Salamon, 1964).

Crouch (1976) developed a numerical technique for calculating displacements and stresses due to complicated excavation geometries based on the "displacement discontinuity method". This method is able to model the behavior not only of subsidence but also of geologic discontinuities, e.g. faults and joints, in a realistic fashion. For example, it is possible to model the opening and closing of a joint or slipping of a fault as the excavation is enlarged. Simulation of the mining sequence can be handled in an efficient way by changing the boundary conditions, i.e. simply by changing some of the algebraic equations. In particular, multiple seam mining at different levels can be formulated in the numerical scheme. The numerical treatment of the problem is simplified, for illustration purposes, if the beginning and end points of each seam are assumed to have the same x-coordinate, as shown in Figure 3-3. The induced normal and shear stress components are given by (Crouch, 1976):

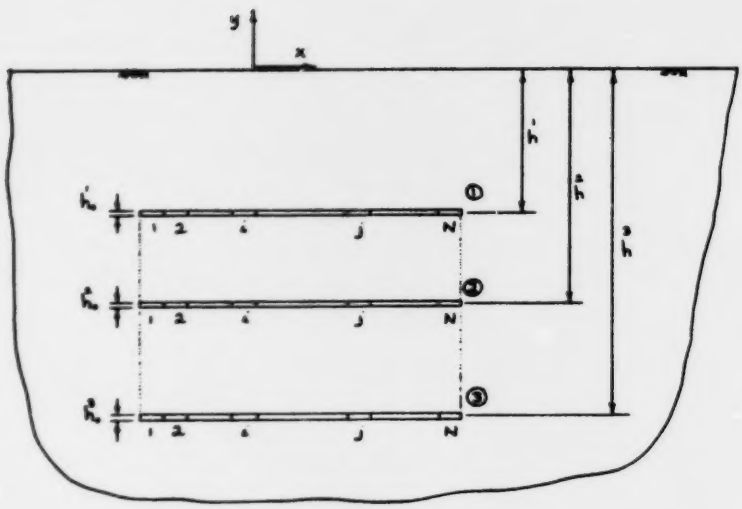

Figure 3-3. Schematic Layout for Displacement Discontinuity Solution to a Multi-Seam Problem. (After Crouch, 1976).

$$(\sigma_s^i)^\ell = \sum_{m=1}^{M} \left\{ \sum_{j=1}^{N} (A_{ss}^{ij})^{\ell m} (D_s^j)^m + \sum_{j=1}^{N} (A_{sn}^{ij})^{\ell m} (D_n^j)^m \right\}$$

$$(\sigma_n^i)^\ell = \sum_{m=1}^{M} \left\{ \sum_{j=1}^{N} (A_{ns}^{ij})^{\ell m} (D_s^j)^m + \sum_{j=1}^{N} (A_{nn}^{ij})^{\ell m} (D_n^j)^m \right\} \qquad (3\text{-}5)$$

where: $(\sigma_n^i)^\ell$ = induced normal stresses at element i of seam l.
$(\sigma_s^i)^\ell$ = induced shear stresses at element i of seam l.
$(D_n^i)^\ell$ = closure at element i of seam l.
$(D_s^i)^\ell$ = ride at element i of seam l.
$(A_{ss}^{ij})^{\ell m}$, $(A_{sn}^{ij})^{\ell m}$, $(A_{ns}^{ij})^{\ell m}$, $(A_{nn}^{ij})^{\ell m}$ are the influence coefficients.
N = number of elements/seam.
M = number of seams.

The influence coefficients are derived using the theory of elasticity and assuming the overburden and seam have different material properties. The derivations of these coefficients are outside the scope of this report but details are well covered by Crouch (1976).

A system of 2xNxM simultaneous, linear equations can be developed from Equation (3-5) by considering the boundary conditions at each element. Solving these sets of equations provides the solution to the problem. The displacement and stress at an arbitrary point (x,y) in the semi-infinite half space may be computed as linear combinations of the displacement discontinuity components $(D_s^j)^\ell$ and $(\hat{D}_n^j)^\ell$. Influence coefficients for this calculation are given by Crouch (1976) and will not be duplicated here.

The "displacement discontinuity" method can model subsidence and can include the effects of slip along a seam or fault during the course of mining as well as dealing with multiple seam simulation. Running the program is relatively simple and cheap. However, the numerical procedures for handling non-linear behavior are still in the development stage. Further research is required for the mixed boundary value problem, i.e. the case where a combination of traction or displacement can be prescribed everywhere on the mined element. Since the rock mass (other than the mine seam) is treated as homogeneous, this method is subject to the same limitations as the closed-form solutions described in Section 3.1.

$$(\sigma_s^i)^\ell = \sum_{m=1}^{M} \left\{ \sum_{j=1}^{N} (A_{ss}^{ij})^{\ell m} (D_s^j)^m + \sum_{j=1}^{N} (A_{sn}^{ij})^{\ell m} (D_n^j)^m \right\}$$

$$(\sigma_n^i)^\ell = \sum_{m=1}^{M} \left\{ \sum_{j=1}^{N} (A_{ns}^{ij})^{\ell m} (D_s^j)^m + \sum_{j=1}^{N} (A_{nn}^{ij})^{\ell m} (D_n^j)^m \right\} \qquad (3\text{-}5)$$

where: $(\sigma_n^i)^\ell$ = induced normal stresses at element i of seam l.

$(\sigma_s^i)^\ell$ = induced shear stresses at element i of seam l.

$(D_n^i)^\ell$ = closure at element i of seam l.

$(D_s^i)^\ell$ = ride at element i of seam l.

$(A_{ss}^{ij})^{\ell m}$, $(A_{sn}^{ij})^{\ell m}$, $(A_{ns}^{ij})^{\ell m}$, $(A_{nn}^{ij})^{\ell m}$ are the influence coefficients.

N = number of elements/seam.

M = number of seams.

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3.4 MECHANISTIC MODELS

3.4.1 Void Volume Method

The critical features that govern subsidence development are discussed in Section 1. A "simple" framework that combines the actions of a discrete deformational mechanism and collapse mechanism has been presented by Munson and Benzley (1980). They postulated that progressive roof collapse and time-dependent deformation or recompaction are the primary modes that influence subsidence. The governing equation takes the following form:

$$S = aV \tag{3-6}$$

where: S = subsidence volume
a = subsidence parameter
V = mined volume

The term, a, consists of a time-independent bulking parameter (a_o) and some time-dependent factors. The time-dependent terms will not be discussed in this report, however, it is interesting to examine the bulking parameter, a_o. This parameter is related to the volume fraction of void which remains underground; the initial volume fraction of void is given by $1-a_o$. According to Munson and Benzley (1980), it is possible to construct a void volume curve which depends on the ratio b/h, where b is a fractured block dimension and h is a height through which the block falls. However a detailed development of this curve is not given. The curve is defined as the void volume distribution function and controls the resulting subsidence at the surface. Further details in applying this curve to obtain surface subsidence have not been clearly stated; but, it is important to realize that the physical parameters required, e.g. characteristic block dimension, cannot be determined theoretically and it is difficult to derive an appropriate void volume distribution function for a specific problem. The application of the void volume method is therefore restricted.

The difficulties in modeling specific, non-elastic mechanisms involved in subsidence led to the development of the void volume method. However,

3.5 DISCUSSION

Clearly, there are limitations and difficulties in applying any of the above methods. One obvious advantage which is common to all analytical/numerical methods, as opposed to empirical methods, is that some physical mechanisms are incorporated in the equations, thus, providing an insight into the complex processes which occur during subsidence. In selecting a method (or methods) for use in Cape Breton it is also important to consider such factors as flexibility, ease of use, and ability to verify the model.

4 SURVEY OF AVAILABLE COMPUTER PROGRAMS

This section provides descriptions of a number of computer programs used for subsidence prediction. The major emphasis is on programs based on analytical/numerical methods, but a few empirical programs will also be described, since it may be possible to calibrate these for use in the Sydney coalfield.

Although a great many computer analyses of subsidence are reported in the literature, the programs have generally been written specifically for the reported application and are neither well-documented nor publically available. A number of subsidence computer programs are available, upon request, from research groups, particularly universities. However, these programs are generally very poorly documented, are in a constant state of revision, and no support is provided in getting them running and verified. For this reason the number of computer programs included in this survey has been limited. They have been selected to represent the different analysis/numerical methods and because they are reasonably well-documented and are reasonably accessible.

One program, BERRY.3D, was written by RE/SPEC for this project to provide an example of the closed-form elastic solution method. The listing for BERRY.3D is given in the Appendix.

4.1 EMPIRICAL METHODS

4.1.1 SPASID

Program SPASID (U.S. Bureau of Mines, 1983) is based on the influence function method and stands for Subsidence Prediction And System IDentification. SPASID was developed by Pennsylvania State University for the U.S. Bureau of Mines, Denver, and is available from the latter organization.

SPASID is written in FORTRAN 77 and consists of four modules: (1) a preprocessor module that prepares and maintains surface and mine data bases, (2) a subsidence computation module that predicts movements of the surface, (3) a system identification module that is used to generate the optimum influence function parameters from measured data, and (4) a post-processor module for displaying surface movements on a graphics device.

Before the influence function method can be implemented for a given site, the parameters associated with the method must be identified. The parameters can be divided into two categories:

1. The influence function parameters: These are adjustable constants appearing in the influence function as influence factors.

2. Site parameters: These are parameters describing the radius of influence, edge effect and the time lag between extraction and subsidence.

SPASID supports two influence functions: the Knothe function (similar to function 7, Table 2-2) and an influence function derived from transversely isotropic elasticity theory (function 9, Table 2-2).

The parameter or system identification can be accomplished by providing appropriate subsidence data. It follows that a system of measuring and recording surface displacements is a prerequisite for successful application of SPASID. The system identification module will compute the optimal values of the parameters required in Knothe's influence function or the elastic influence function.

The slope of the seam inclination must be less than ten degrees in order to give reasonable prediction. Considerable uncertainty exists about the horizontal movements when seam inclination is greater than this value. The horizontal displacements must also be re-evaluated when experimental data becomes available.

Program SPASID runs on mainframe computers such as IBM, CDC 6400 and HP 1000. The post processor module supports Tektronix 4000 series graphics devices, as well as the CALCOMP style of equipment. SPASID is supported by a comprehensive user's manual and is relatively easy to use.

4.1.2 MINSUBSIM

The program provides subsidence prediction based on European continental experience. MINSUBSIM is based on the profile function method

and refers to MINing SUBsidence SIMulatcr (U.S. Dept. of Energy, 1981). The profile function is calibrated to the graphical method described in the Subsidence Engineers' Handbook (National Coal Board, 1975). The transverse subsidence profile is simulated by:

$$S(X) = B1\ (1 + (X - B2)^2 / (B3^2 \times B4))^{-B4}$$

and the constants B1 through B4 vary according to the width/height ratio. The constants were determined to fit the data points obtained from the graphs given by Shadbolt (1978).

The simulator is based on European experience, and there is no provision for inputting different values for the parameters or for fitting field data to obtain parameter values.

The program was developed by Virginia Polytechnic Institute and State University for the U.S. Department of Energy and is available from the latter organization. The program is supported by a user's manual, is written in FORTRAN and has been designed to be a stand-alone tool or it can be linked with a larger longwall simulation program.

4.1.3 ZONE

A computer model of subsidence has been developed based on the zone area method by Goodman (1980) at the Virginia Polytechnic Institute and State University. A listing of the program is provided in Goodman's thesis.

The program utilizes the influence function and a rectangular grid is established over the area to be investigated. About every grid line intersection, a series of seven annular rings is constructed. The product of the proportional extracted area and zone factor for each zone yields that zone's contribution to the total subsidence. Addition of these products for each of the seven rings results in the subsidence at that point.

For the grid line intersections located over barrier pillars with a width/depth ratio of less than 0.5, a five zone pillar reduction routine is used. Subtraction of the reduction value from the seven zone subsidence value yields the true subsidence value over the pillar.

The input data consists of a data block describing the geometry of the workings and boundaries of the pillars. Seven zone factors and a constant

describing the non-linearity relationship between zone factor and proportional extracted area are required as well as the angle of influence. The surface topography and the coal seam can be either level or inclined.

This program was used by Karmis et al (1981) to study subsidence in the Appalachian coalfields and they concluded that it is capable of realistic subsidence prediction.

Program ZONE is written in FORTRAN 77 for implementation on mainframe computers, although a microcomputer version is also available (RE/SPEC, 1985).

4.2 FINITE ELEMENT METHOD

4.2.1 GEOROC

GEOROC is a major non-linear finite element program designed specificially for underground analyses. GEOROC stands for GEOtechnical ROck mechanics analysis with Creep. It predicts stresses, displacements and failure for complex underground openings and has a broad range of applications encompassing mining hard and soft rocks, storage caverns in salt, nuclear waste repositories and oil and gas. GEOROC was developed by RE/SPEC Ltd. and is described by Fossum et al (1985).

GEOROC is based on the finite element method (Zienkiewicz, 1977) and uses eight-noded isoparametric quadrilateral elements. It is written in FORTRAN 77 and for implementation on mainframe computers. It can solve problems incorporating two-dimensional (plane strain and plane stress) or axisymmetric geometry and can include isotropic elastic, transversely isotropic elastic, visco-elastic and plastic material properties. Complex stratigraphy and mined openings, including multiple-level mining, can be modeled by appropriate mesh design and choice of material types. Initial and sequential excavations can be incorporated as well as time-dependent material properties. The in-situ stress field can be non-uniform and is based on body forces or overburden weight and a lateral earth pressure coefficient.

GEOROC is well suited to subsidence analysis in many respects. It can model complex stratigraphy and multiple seam mining as well as elastic, and non-linear rock properties. It can also incorporate non-uniform stress fields and predict failure by several criteria.

Limitations of GEOROC are that it cannot directly model block caving, such as occurs in the roof of longwall coal mines, or bed separation. Both these factors are considered to be important mechanisms in the development of subsidence troughs.

All general finite element programs, including GEOROC, are relatively cumbersome to use and expensive to run compared to special-purpose programs such as ZONE and SUBSID-B. Since GEOROC has been written specifically for underground applications it is more streamlined than other FEM programs such as ADINA and should, therefore, be preferred for subsidence analyses.

4.2.2 SUBSID-B

SUBSID-B is a non-linear, two-dimensional, finite element program developed for static mining subsidence. This program has been designed to determine subsidence and stresses over single seam mines with room and pillar or longwall type of extraction. It was originally developed by Ratigan and Goodman (1980) in FORTRAN 77 for mainframe implementation. SUBSID-B was subsequently converted for use on Hewlett Packard series 200 microcomputers by RE/SPEC Ltd. (Paslawski and Tammemagi, 1983).

The program utilizes engineering beam theory and a simplified FEM to determine subsidence due to underground mining. Three basic element types are defined to represent overburden, pillars and rooms. The behaviour of the overburden elements is governed by slope deflection equations of classical beam theory with shear deflections included. Pillar and room elements are assumed to be rigidly attached to a fixed base. Room elements can be either empty, i.e. having zero compressive strength or backfilled with specified materials. Vertical stresses are assumed to be constant along the horizontal plane and no tension is allowed. The overburden elements exhibit only linearly elastic constitutive relationships. However, behavior of both room and pillar elements can be assumed to be either linear or non-linear. Non-linear behavior is modeled by the S-shaped curve of Figure 4-1 which is typical of soft rock behaviour (e.g. coal).

Limitations of the program are that it cannot account for inclined bedding (dipping seams) or multiple seams. The model assumes that the mining seam is connected to a rigid base. It does not model block caving and bed

RSL DWG 888-83-04

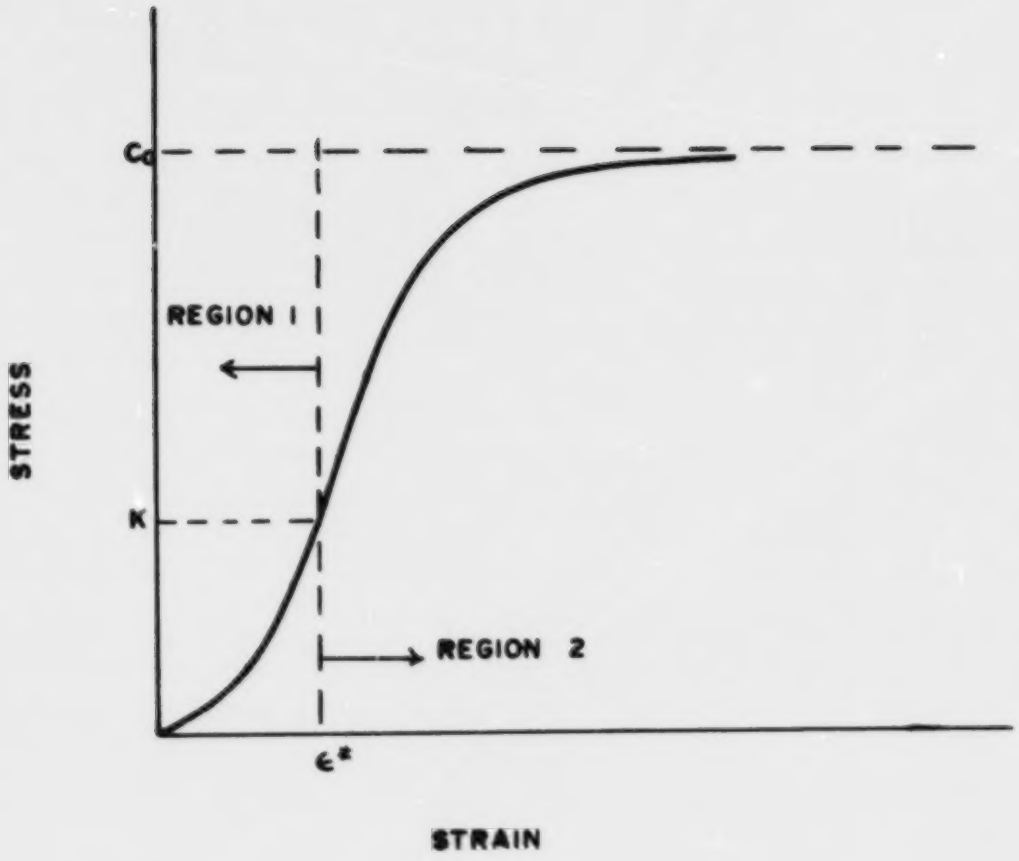

Figure 4-1. Non-Linear Stress-Strain Response of Room and Pillar Elements in SUBSID-B.

separation, both of which may be significant in the development of subsidence troughs. However, SUBSID-B is supported by a comprehensive user's manual, is easy to use, and has been successfully applied to studying subsidence over potash mines in Saskatchewan (Bawden, 1984).

4.2.3 ADINA

ADINA is a large, multi-purpose program which is applicable to virtually all areas of engineering mechanics and may be considered on the forefront of finite element packages available today. It is an extremely complex program (more than 57,000 lines) which is capable of linear, non-linear and dynamic analysis of 2- and 3-dimensional problems. A large number of material models can be used, including elastic, plastic and viscoelastic solids and fluids. ADINA is available from ADINA Engineering Inc., Watertown, MA.

In many respects ADINA has the same features as GEOROC when applied to underground excavation analysis. It has a similar range of material models, similar geometrical flexibility, and allows elements to be 'excavated' during time-dependent non-linear analyses using a "birth and death" option. The ability to handle 'transition' elements, (elements with between 4 and 8 nodes) permits economical solution by minimizing the number of nodes in regions of the model where stress gradients are low. This feature is valuable in subsidence problems where the region modeled must be very large.

ADINA is widely used in many areas of structural and mechanical engineering. Although its use for subsidence prediction is not mentioned in the literature, it has been applied to other mining/rock mechanics problems (e.g. Langer and Stockmann (1985), Chan and Nielsen (1985)). As with GEOROC, ADINA is better able to model the local rock behavior associated with small excavations than the large scale problem of subsidence in which the material behavior is more complex.

ADINA is written in FORTRAN 77 for implementation on mainframe computers.

4.2.4 Other FEM Programs

The finite element method has become a standard tool in virtually all

branches of engineering and, thus, a large number of FEM programs is available. For example Dahl and Choi (1973) and Ishijima and Isobe (1973) describe detailed analyses of subsidence using programs which simulate failure and also (for the former) are three-dimensional. Program BMINES was developed by Agbabian Associates (California) for the U.S. Bureau of Mines specifically to address some of the special requirements in modeling a rock mass. Although many programs are private research tools, there are also many commercially available programs. Some of the better known ones are MARC, ANSYS, ADINA, NASTRAN, ABACUS and STARDYNE. These are all large, complex programs with many non-linear options and element types.

4.3 CLOSED-FORM ELASTIC PROGRAMS

4.3.1 BERRY.3D

BERRY.3D is a three-dimensional, closed-form solution program for analyzing surface strain and subsidence due to underground coal mining based on the theoretical model of Berry (1960) and Berry and Sales (1961, 1962). Besides performing the strictly analytical approach, the empirical model given by Avasthi and Harloff (1982) has also been incorporated in the program. The improved model performs empirical transformation and adjustments which reconcile NCB field data with theoretical calculations.

The rock mass is assumed to exhibit homogeneous, linearly elastic and transversely isotropic material properties. In addition, the depth of excavation is large compared with its width. The thickness of the excavated material is comparatively small relative to depth and, therefore, the thickness is considered infinitesimal. The solution only applies to the case of complete closure of the excavated panel which must be parallel to the ground surface.

The input data required is straightforward. Information regarding the mine geometry includes the half width and half length, depth and thickness. The user can specify the output grid lines and size. Material properties are related to two elastic constants, k_1 and k_2 (see Section 3.2). The code can operate in two options: the analytical approach or the modified (empirical) three-dimensional approach. The program outputs the subsidence and horizontal strain at the surface in the x and y directions. The program can

be extended for multiple cavities in one level using the superposition technique. The solution to the case of a steeply dipping coal seam has been demonstrated by Avasthi and Harloff (1982).

BERRY.3D was programmed specifically for this project to provide an example of the closed-form elastic method of subsidence prediction. The program is very easy to use. It is available for the Hewlett Packard series 200 as well as IBMpc compatible microcomputers.

4.4 BOUNDARY INTEGRAL

4.4.1 MSEAMS

Program MSEAMS is based on the displacement discontinuity method, which is a form of "boundary element" or "boundary integral" method. MSEAMS means Multiple SEAM Simulation.

The program performs subsidence calculations for as many as five parallel seams. It can be applied to a variety of plane strain, stress analysis problems in mining as well as for subsidence prediction. The program assumes that the rock mass is a homogeneous, transversely isotropic, linearly elastic material, with planes of elastic symmetry and seams parallel to the Earth's surface. In addition, the seams are assumed to be negligibly thin compared to their lateral extent.

The program requires that the seams be input as a number of discrete elements. There are two restrictions for multiple seam problems: (1) no seam can be closer than one element length to the Earth's surface and (2) any two seams must be more than one element length apart. The seam element can be either mined or unmined. Other input information consists of Young's moduli and Poisson's ratio in three directions and three planes as well as the 'cross-bedding' shear modulus describing the behaviour of overburden mass.

The results are output in two main parts: an on-seam solution and an off-seam solution. Besides compilation of displacements, stresses and strain for off-seam points, the program also outputs the closure (y-component of displacement) and the ride (x-component of displacement) of the seam. Therefore, the convergence of the roof and floor can be evaluated.

MSEAMS was developed by Crouch Research Inc., St. Paul, Minnesota, and is written in FORTRAN for use on IBMpc compatible microcomputers. It is easy to use and is supported by a user's manual.

4.4.2 NFOLD

Program NFOLD was developed to analyze the changes in stresses and displacements which result from underground mining. The program analyzes problems in three dimensions based on the displacement discontinuity numerical technique. It can model multiple ore bodies with any dip (but uniform strike) and large and complicated mining patterns. Sequential mining can be simulated by progressively removing elements. Faults are modelled in a similar manner, with each element having the thickness of the fault.

While the rock mass away from the seam must be assumed to be an infinite or semi-infinite elastic material, various types of load-deformation behaviour can be modeled for the unmined rock within the seam. If the peak strength of the unmined rock within the seam is known, the unmined rock can be modeled elastically until this peak load is reached and, beyond this, a falling load-deformation curve can be used. The influence of confinement on strength of rock mass can also be modeled. The shear behavior of faults is specified according to the Mohr-Coulomb criterion.

The program is capable of modeling mine fill and a complementary computer program, NFOLPL, was developed to plot stress and displacement contours from the results of NFOLD analysis.

NFOLD was developed by Golder Associates Ltd., Vancouver, and is written in FORTRAN 77 for implementation on mainframe computers. It is based on the initial formulations of Starfield and Crouch (1973) and has been used in a number of mining case history studies (e.g., Dunbar and Hammett, 1983).

4.5 FINITE DIFFERENCE METHOD

4.5.1 STEALTH

The explicit finite-difference codes, STEALTH, were developed by Science Applications Incorporated for the Electric Power Research Institute. These

codes are based on the published research of the Lawrence Livermore National Laboratory and Sandia National Laboratory. They have been used for many years to model nuclear containment and cratering experiments and recently, have been applied to modern civil and geotechnical problems. Both one and two-dimensional versions of STEALTH are available to the public while the three-dimensional version will be available soon.

Owing to the explicit nature of the finite difference formulation, the codes allow complicated non-linear large strain behavior, for example subsidence problems, to be modeled in a relatively straightforward fashion. Highly sophisticated models such as jointslip and shear fracture behavior can be incorporated in the STEALTH codes and research has been actively carried out by Science Applications Incorporated to investigate the following mechanistic models:

1. fracturing
2. joint-slip
3. transient and steady-state pore pressure effects

STEALTH has been used to model underground stress and displacement associated with nuclear waste disposal into bedded salt formations (Morgan et al., 1981), and subsidence related to mining (Trent, 1981). It is similar to the major finite element programs in terms of its wide range of capabilities and complexity.

4.6 DISCUSSION

A wide range of factors must be considered when making a decision as to which computer program should be used for subsidence analysis. These factors may be divided into two general groups: the first describes the technical capabilities of the specific program; the second includes other details which have been arbitrarily termed as 'administrative' considerations.

A number of specific factors for selecting and/or ranking subsidence computer programs are listed in Table 4-1. The technical factors in Table 4-1 must be assessed with regard to how they apply to the geology and mine layout which is to be analyzed and the administrative factors must be assessed in view of staff availability, budgets and type of computer on which analyses are to be done.

Table 4-1. Factors to be Considered in Ranking Subsidence Computer Programs

A. Technical Factors

Ability to model:

- multiple seams
- dipping seam(s)
- multiple panels
- complex panel geometry
- topography
- transversely isotropic rock properties
- laterally inhomogeneous rock properties
- strata bending
- roof caving
- bed separation } analytical
- tensile cracking
- joint slip
- does it have a good data base? } empirical

B. "Administrative" Factors

- user-friendliness
- documentation
- cost (availability)
- cost to run (labor and computer)
- output/graphics
- on what computers does it run?

5 ANALYSIS OF CASE HISTORY

5.1 INTRODUCTION

Six of the computer programs described in Section 4 were selected to be tested on a hypothetical case history supplied by CANMET. The programs were selected primarily on the basis that a good cross-section of different available methods would be tested. Another criterion was that all programs had to be run by RE/SPEC staff, rather than independent groups, so that various characteristics of the programs such as flexibility and ease of use could be evaluated. The six programs are ZONE, BERRY.3D, SUBSID-B, GEOROC, ADINA and MSEAMS; in addition the NCB graphical method was also utilized.

5.2 DESCRIPTION OF CASE HISTORY

The hypothetical case history was provided by CANMET and was chosen to be representative of a single longwall coal mining panel located in strata of the Sydney coalfield. A schematic layout is shown in Figure 5-1. The coal seam is located under 198 m of typical coal measure strata and is 2 m thick. The panel is 100 m wide, 2000 m long and 2 m in height. The elastic rock properties for each of the rock types are listed in Table 5-1. These were provided by CANMET and are based on laboratory measurements on core samples. It is assumed that the materials are homogeneous, isotropic and contain no significant fractures or faults. It is also assumed that the geologic strata and, hence, the longwall panel are horizontal and that the in situ stress surrounding the panel is isotropic and caused by the lithostatic pressure exerted by the overlying rock mass. It is seen that this case history is very simple and avoids many of the complexities associated with real mining situations.

5.3 RESULTS

5.3.1 Analyses With ZONE

The zone area method was applied to the case history in three separate analyses using program ZONE. In all three cases the geometry of the case history and the locations of the desired output points were input in a

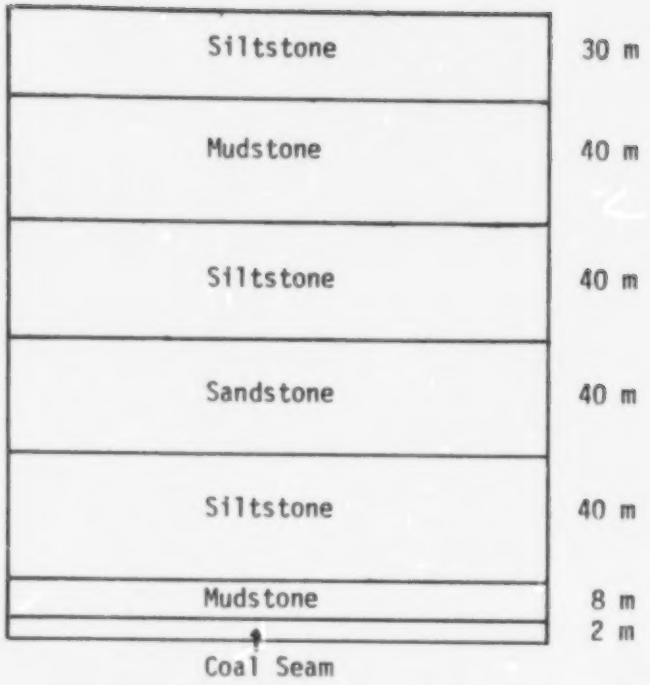

Figure 5-1. Schematic Diagram showing Stratigraphy and Panel Dimensions for the Hypothetical Case History.

Table 5-1. Rock Properties for the Hypothetical Case History

	Unconfined Compressive Strength (MPa)	Tensile Strength (MPa)	Youngs Modulus (GPa)	Poisson's Ratio	Density (kg/m^3)
Sandstone	66.0	10.5	20.0	0.20	2600
Siltstone	50.0	8.0	10.0	0.18	2600
Mudstone	20.0	5.0	4.0	0.13	2600
Coal	21.0	2.0	4.0	0.25	1300

straight-forward fashion. Since ZONE is based on an empirical method no rock properties were required. However, three sets of different zone factors were used representing subsidence from the U.K., eastern U.S.A. and western U.S.A. (see Table 2-3).

The resulting subsidence profiles are plotted along the horizontal axis corresponding to the ground level in Figure 5-2. The curve using the U.K. zone factors extends the furthest to about 110 m, while the eastern U.S. experiences the least radius of influence, i.e. approximately 80-90 m from the centre of the panel. The magnitudes of maximum subsidence vary from 1.2 m, for the case of the western U.S. to about .9 m in the eastern U.S. Using the U.K. zone factors, subsidence of about 1 m is calculated. The point of inflection for the three curves ranges from 30 m to about 40 m from the centre line. The general shape of the curves corresponds to typical subsidence profiles given by the National Coal Board.

5.3.2 Analyses with BERRY.3D

The input data for BERRY.3D is straight-forward. The program prompts the user for information regarding the geometry of the panel and at what points output is required. In addition, two elastic constants, k_1 and k_2 have to be specified to describe the transversely isotropic properties of the overburden. These constants become unity for an isotropic medium such as the case history.

Two analyses of the case history were performed with BERRY.3D. In the first the strictly analytical approach was used with $k_1 = k_2 = 1$. The "modified" (empirical) three-dimensional analysis was also used with $k_1 = 3.965$ and $k_2 = 1.79$, as recommended by Avasthi and Harloff (1982). These two curves together with the one derived from the National Coal Board graphs are plotted in Figure 5-3. The "modified" (empirical) three-dimensional theory shows reasonable agreement with the NCB curve as would be expected, while the analytical closed-form solution results in a smaller maximum subsidence of about 0.6 m. The strictly analytical solution yields a much broader profile than the modified or NCB curves. The points of inflection as shown in Figure 5-3 for the analytical and modified curves are about 40 m and 120 m respectively.

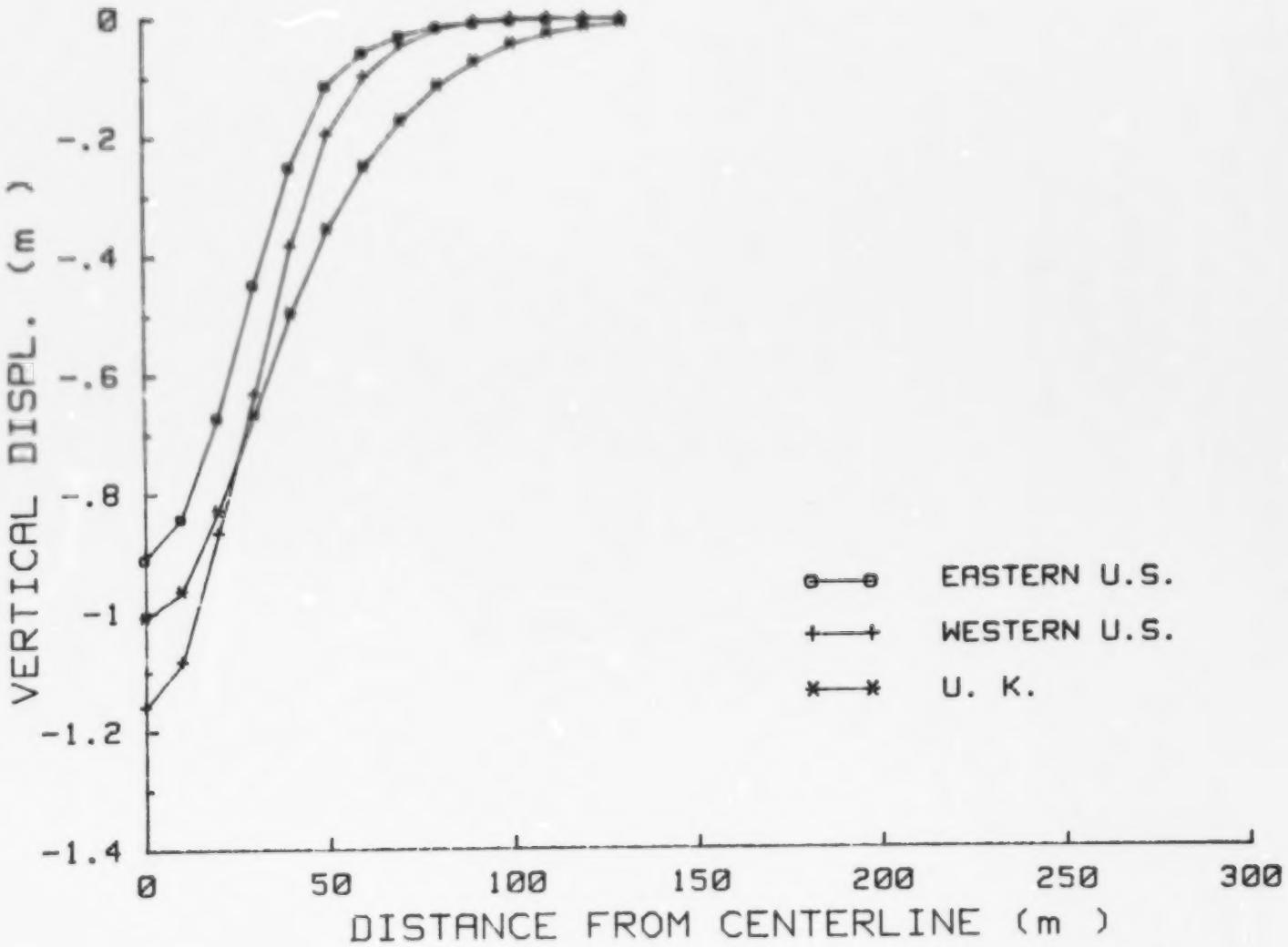

Figure 5-2. Plot of Surface Subsidence Against Horizontal Distance Using the Zone Method with (1) U.K. Zone Factors, (2) Eastern U.S. Zone Factors and (3) Western U.S. Zone Factors.

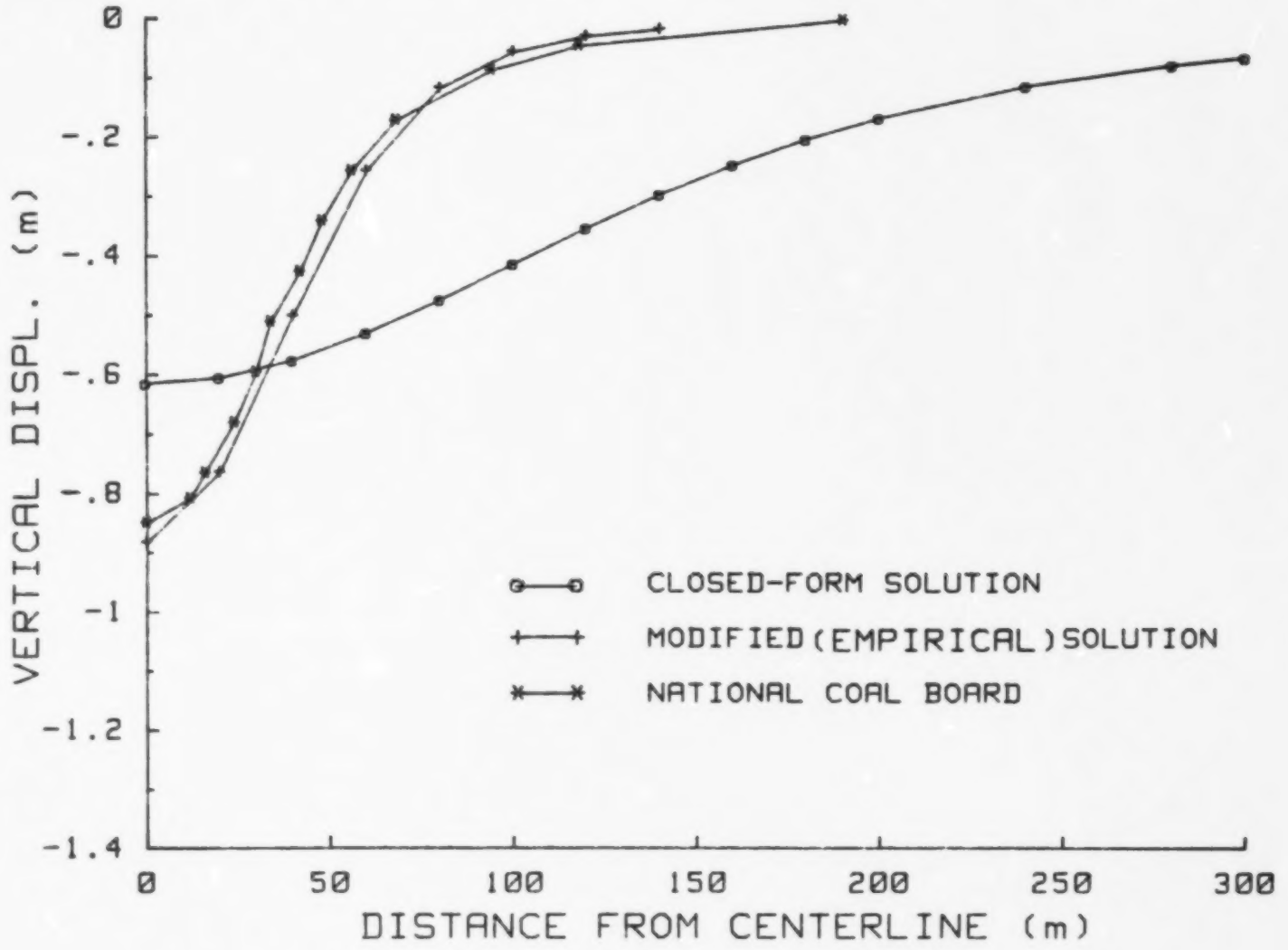

Figure 5-3. Plot of Surface Subsidence Against Horizontal Distance Obtained from (1) National Coal Board Graphical Method, (2) Berry and Sales Closed-Form Solution and (3) "Modified" Closed-Form Method.

5.3.3 Analysis with SUBSID-B

The layout of the finite element grid for the case history analyses with program SUBSID-B, is shown in Figure 5-4. (Note that the height of the seam is exaggerated). The overburden is divided into 36 elements with a height of 198 m. There are 10 and 26 room and pillar elements respectively. The program assumes homogeneous isotropic overburden and therefore an equivalent modulus of elasticity representing the total overburden is required. This equivalent Young's modulus is calculated using an averaged strain assumption as suggested by Berry (1978, Equation 20). The Young's modulus of a uniform overburden equivalent to that shown in Figure 5-1 was calculated to be 7920 MPa. A Poisson's ratio of 0.17 was used for the equivalent uniform overburden. The properties of coal were assumed to be linear and elastic (as shown in Table 5-1) and analysis was performed without any "backfill" for room elements.

The results of the analysis are shown in Figure 5-5. It is seen that program SUBSID-B predicts a maximum subsidence of approximately 2 cm with the area of influence extending to about 90 m. The latter value is comparable to that predicted by the NCB or zone methods (see Figures 5-2 and 5-3) but the maximum subsidence is substantially less (by a factor of about 50!).

5.3.4 Analysis with GEOROC

The continuum under consideration was divided into a mesh of discrete rectangular elements interconnected at their nodal points (Figure 5-6). This model has 380 elements and 1219 nodal points. Mesh refinement is finest in the region close to the panel and becomes progressively more coarse towards the vertical and lateral boundaries. There is a plane of symmetry along the left vertical boundary and the right vertical boundary was placed sufficiently far away so it would not influence the results. The problem was run using the plane strain assumption and linear elastic and time-independent rock properties. The exact strata for the overburden and the seam as shown in Figure 5-1 were modeled and the rock properties given in Table 5-1 used. The rock mass lying below the coal seam was modeled with elastic moduli obtained by averaging the values in Table 5-1.

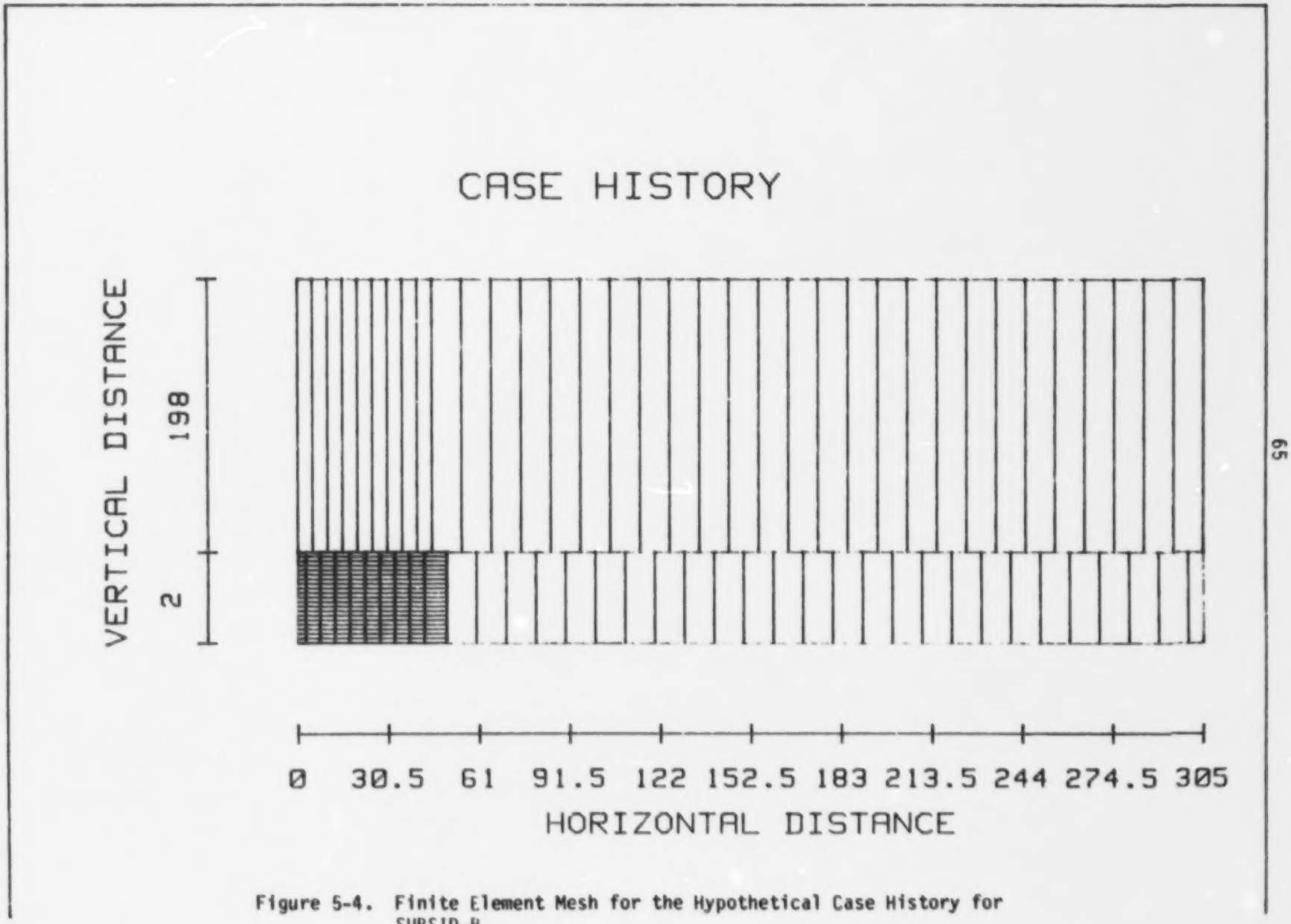

Figure 5-4. Finite Element Mesh for the Hypothetical Case History for SUBSID-B.

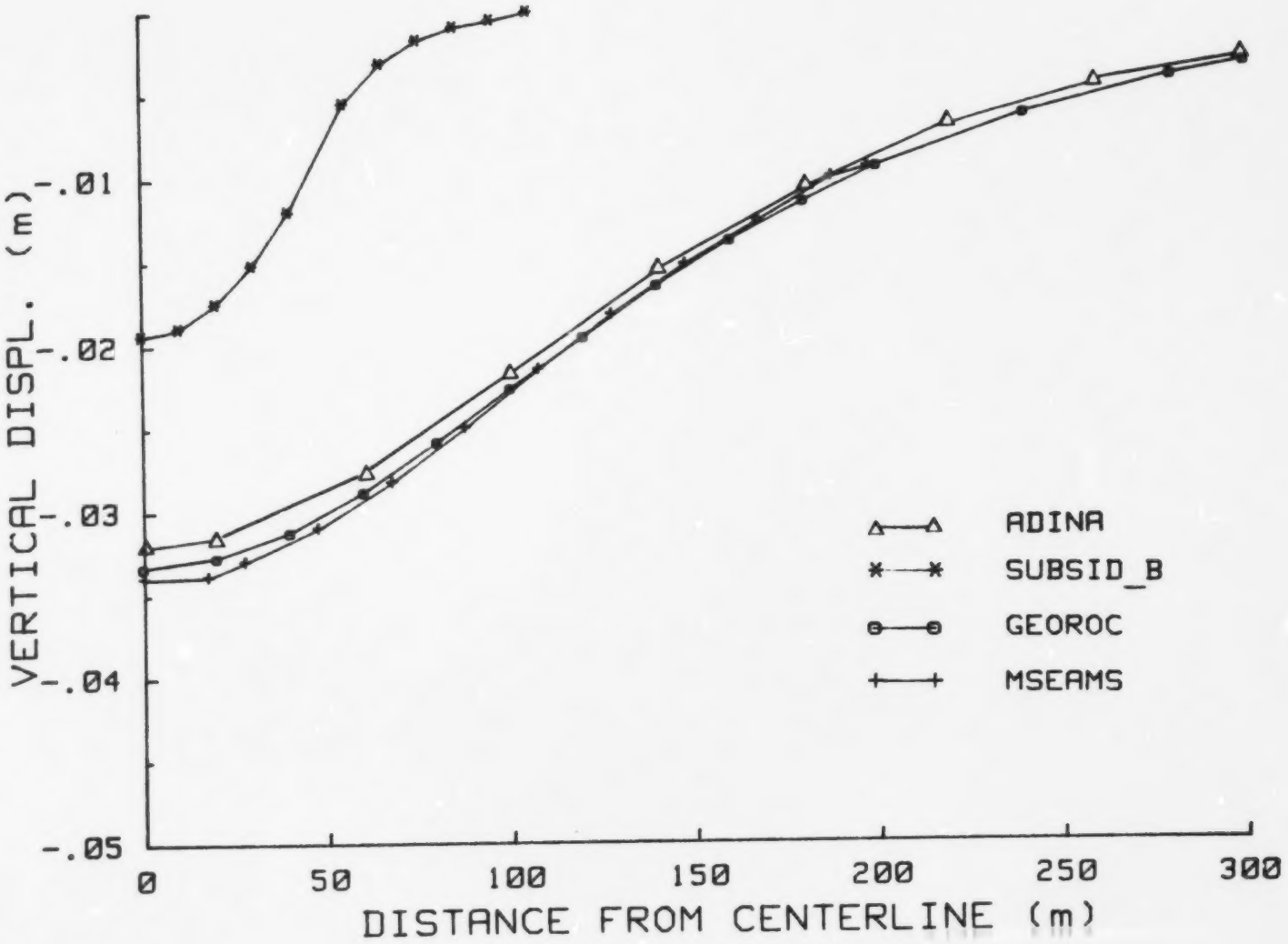

Figure 5-5. Surface Subsidence Plotted Against Horizontal Distance for (1) SUBSID-B, (2) GEOROC, (3) ADINA and (4) MSEAMS.

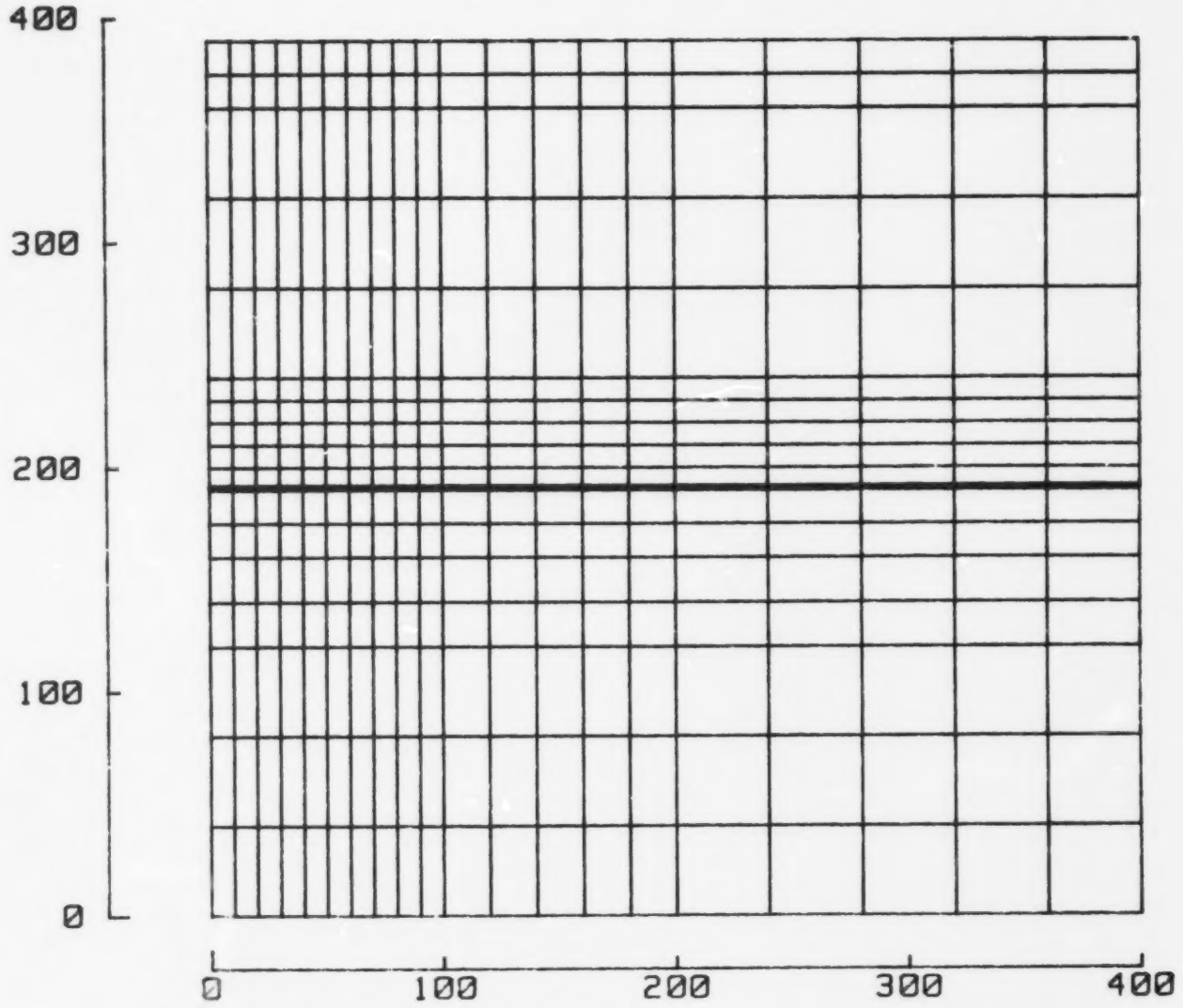

Figure 5-6. Finite Element Mesh of the Hypothetical Case History for GEOROC.

Figure 5-5 shows the subsidence results obtained from GEOROC. A maximum subsidence of about 3.3 cm occurs over the centre of the panel and the area of influence extends to over 300 m. As for SUBSID-B, the amount of subsidence predicted by GEOROC is considerably less than that predicted by the empirical methods (ZONE, NCB). The panel had a maximum vertical closure of 13.1 cm or about 7% total closure.

5.3.5 Analysis with ADINA

As discussed earlier, ADINA is a large non-linear finite element package designed to be applied to as wide as possible a range of problems in engineering mechanics. This being so, it is to be expected that its application to any particular and unusual problem will not be straightforward. Thus, a number of minor difficulties arose in modeling the subsidence case history.

The finite element mesh used in the analysis by GEOROC proved too large for the available array dimensions within ADINA and a more efficient mesh was generated, as shown in Figure 5-7. In this mesh eight-noded elements are only used around the mine opening where high stress concentrations dictate the use of higher order elements. Five, six and seven noded transition elements are employed so that elements remote from the opening can be larger and need only have four nodes. The result is a mesh with 172 elements and 272 nodes which should be capable of similar accuracy to the mesh in Figure 5-6.

The strata are exactly as given in Figure 5-1 with a mirror image of the same stratigraphy below the seam. Since the number of nodes per element varies between four and eight, automatic mesh generation was not possible. Hand calculation of nodal coordinates and manual entering of these and the element topology table into a data file resulted in a period of data "de-bugging".

The available options for applying load to a linear elastic model do not include a hydrostatic in-situ stress field for two dimensional elements, although this option is available for the no-tension plasticity material model. As a result it was necessary to calculate the in-situ stresses at the depth of the seam and apply the negatives of those stresses to the boundary of the excavations. A weightless model with the negative of the in-situ stress

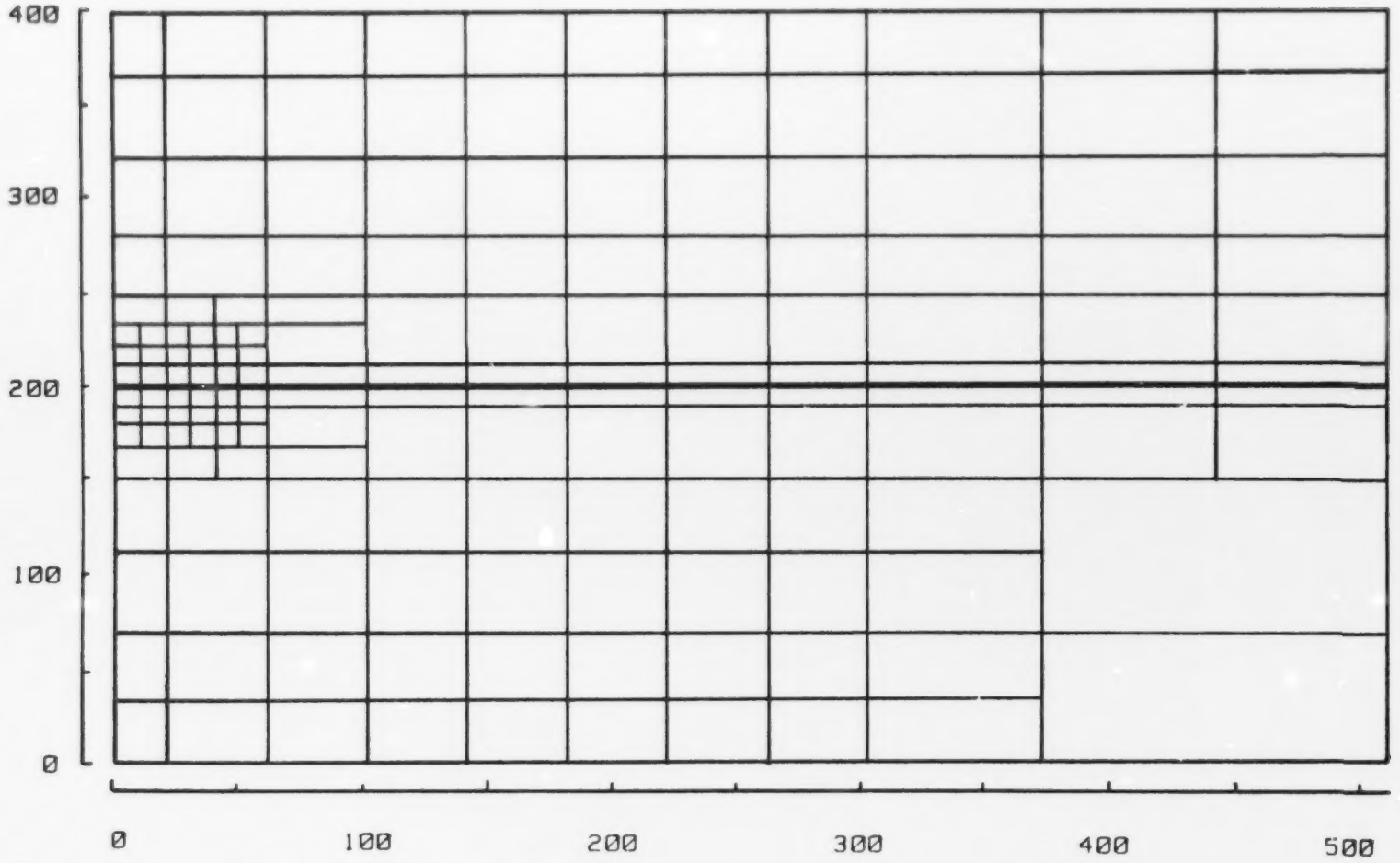

Figure 5-7. Finite Element Mesh of the Hypothetical Case History for ADINA.

applied to the cavity boundary is equivalent to a model with hydrostatic in-situ stress with zero stresses on the cavity boundary and yields identical displacements. True stresses in the deformed model are the sum of the original in-situ stresses and the stresses induced by the applied boundary stresses.

The surface displacement profile calculated for the ADINA model is shown in Figure 5-5. Agreement with the results produced by MSEAMS and GEOROC is seen to be extremely good. The slight reduction in maximum subsidence as compared to GEOROC is likely to be a result of the rather coarser mesh used in the upper portion of the overburden. Total vertical room closure predicted was 12.5 cm.

5.3.6 Analysis with MSEAMS

In the analysis using MSEAMS the coal seam was divided into 80 elements, each having a width of 5 m. Excavation was simulated by specifying 20 mined elements in the middle of the seam with the rest of the elements remaining unmined. The equivalent elastic modulus for a homogeneous overburden (7920 MPa) was calculated as described in Section 5.3.3 with a Poisson's ratio of 0.17. For the seam element, the elastic properties of coal were specified (see Table 5-1). In general, any off-seam displacements can be evaluated and the off-seam line corresponding to the ground level was specified in this analysis.

The results from the MSEAMS analysis are shown in Figure 5-5. The maximum subsidence (about 3.3 cm) and shape of the trough are virtually identical with that calculated by GEOROC and ADINA. Such a good fit is somewhat surprising since GEOROC included each overburden stratum explicitly whereas MSEAMS used an equivalent homogeneous overburden. There was a vertical closure of the panel of 13.4 cm which represents 7% closure.

5.4 ADDITIONAL ANALYSES

A few additional analyses were performed with parameters different from those specified for the case history. A series of runs were carried out with SUBSID-B to see if the NCB profile for the hypothetical case history could be

matched. To simplify matters it was assumed that both the overburden and coal seam had the same elastic properties. Figure 5-8 shows the subsidence profile obtained using a Young's modulus of 158 MPa and a Poisson's ratio of 0.172; the NCB curve is shown for comparison and it is seen that there is good agreement. The maximum subsidence is approximately 0.8 - 0.9 m with the radius of influence extending about 100 m. To achieve this result the Young's modulus was reduced by a factor of about 50 from that used to produce the curve of Figure 5-5. The purpose of this analysis was to illustrate that a wide range of results is possible using programs such as SUBSID-B, depending on what is used for input data. It does not imply that the NCB results are more correct than the subsidence produced by SUBSID-B, MSEAMS, GEOROC and ADINA for this hypothetical case history.

Two separate problems were analyzed using GEOROC. In the first, an 18 m thick zone of rock in the roof was modeled with a greatly reduced modulus (20 MPa) to simulate the caving of the roof. It was felt that such a crude model could provide a reasonable estimate of actual behavior while avoiding the much more complex task of incorporating non-linear failure and block-slip mechanisms. The results yielded a maximum subsidence of 4 cm which is only marginally more than shown in Figure 5-5. It was not possible to match the NCB profile by this method of caved-zone simulation.

In the second series of analyses with GEOROC, the entire geologic sequence, including the coal seam, was treated as a homogeneous, elastic material and the Young's modulus was varied until the subsidence profile approximately matched that of the NCB. Figure 5-9 shows the results for a modulus of 260 MPa. It is seen that the maximum subsidence has been well matched whereas the extent of influence has not. For this particular analysis a total panel closure of 3.7 m resulted which exceeds the height of the panel. This is physically unreasonable and suggests that for the given value of modulus, less subsidence would probably have occurred if the closure had been constrained to 2.0 m. In this case, it is likely that the profile would have been similar to that produced by BERRY.3D (unmodified) and shown in Figure 5-3.

5.5 DISCUSSION

The results presented in the preceding sections are clearly of two distinct types. The empirical methods (ZONE, NCB, modified BERRY.3D) produce results

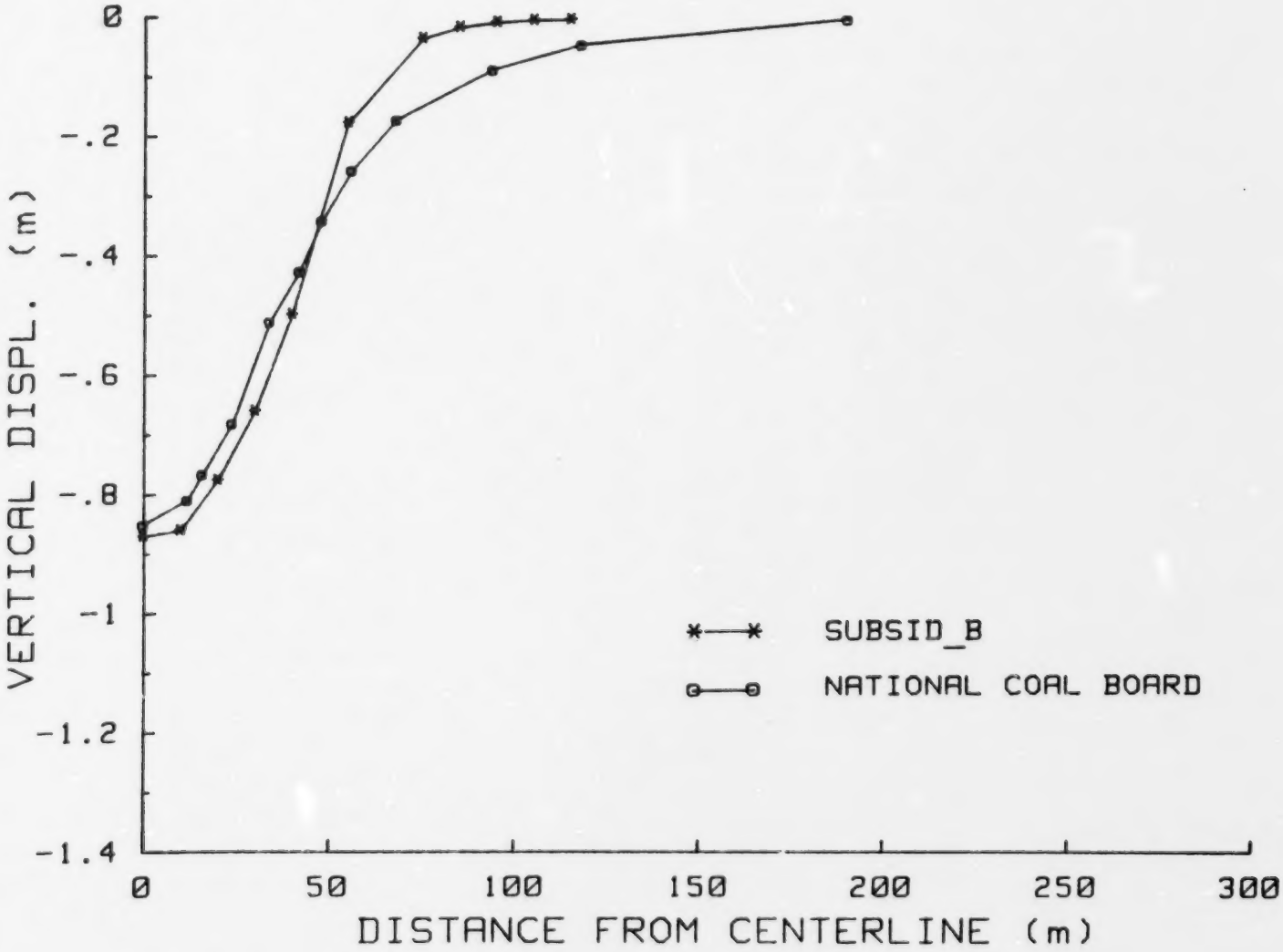

Figure 5-8. Subsidence Results Obtained Using a Reduced Young's Modulus with SUBSID-B and the National Coal Board Graphical Method.

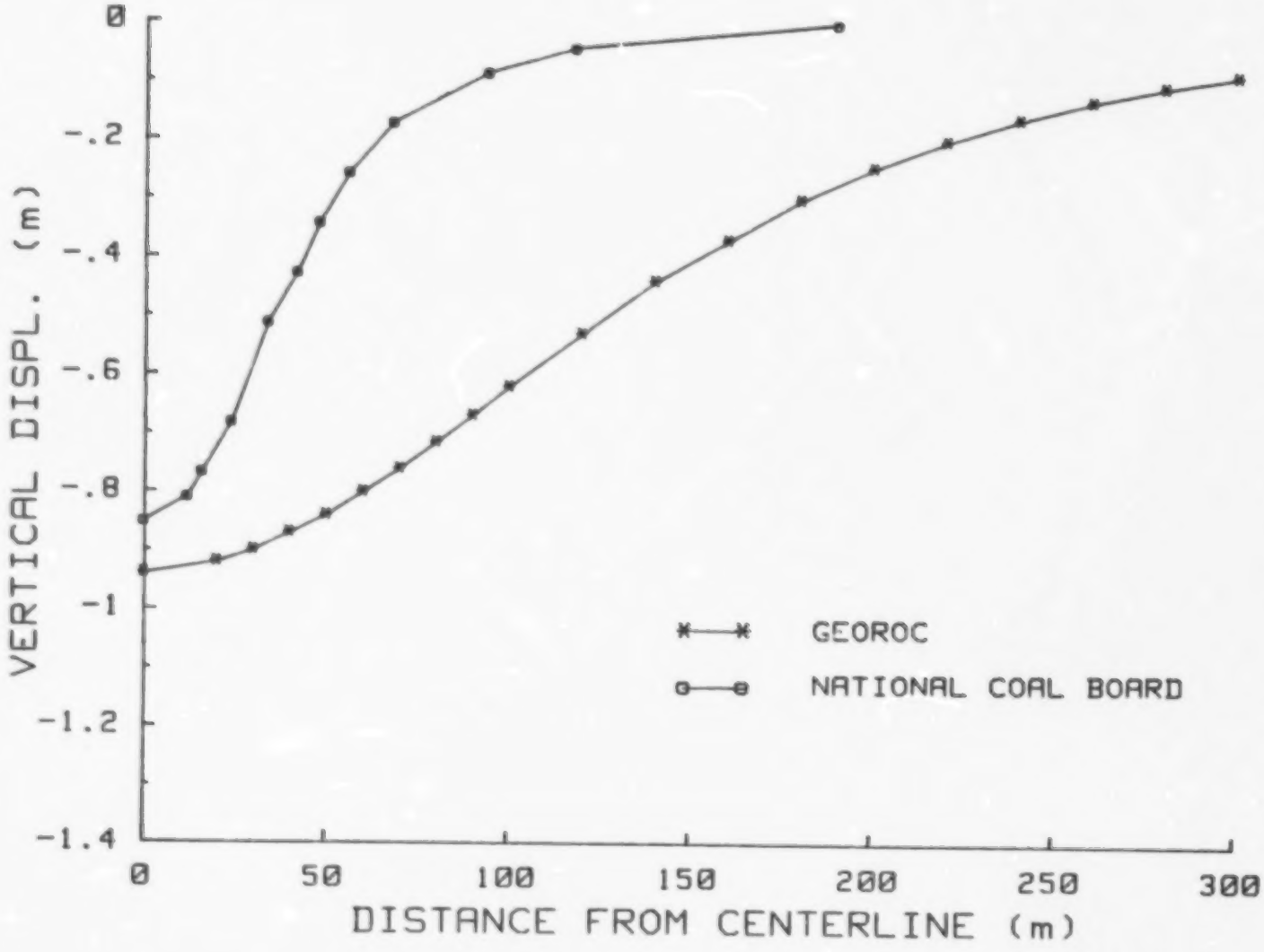

Figure 5-9. Surface Subsidence Obtained Using GEOROC and a Reduced Young's Modulus.

which are consistent with each other and yield a relatively large maximum subsidence (about 1 metre) with a relatively limited extent of influence (approximately 100 m). The analytical methods (SUBSID-B, ADINA, GEOROC, BERRY.3D, MSEAMS) produced results which are not as internally consistent, but which yield a much smaller maximum subsidence (2-7 cm) and larger radius of influence (300 m) than the empirical methods. There are some discrepencies between the analytical results. The much larger subsidence for the BERRY.3D model than the others is probably due to the complete closure of the panel whereas the GEOROC, ADINA and MSEAMS models only undergo about 7% closure. Programs GEOROC, ADINA and MSEAMS produce results which are in excellent agreement. SUBSID-B produces subsidence results which are similar to, but slightly different, from those of ADINA. The cause of this slight discrepancy is not understood.

It is inherently difficult to assess the validity of the results on an absolute scale since a hypothetical case history has been analyzed and no field data are available. However, based on experience it appears that the analytical models, which are based on the theories of elasticity, are overly stiff. Since the analytical methods are relatively consistent with each other it would seem that these computer programs are functioning 'correctly' on a technical basis. The possible discrepancy in results may be due to the fact that:

1. the input data are not representative of field values.

2. the analytical methods only model strata bending and other mechanisms may be involved (see Figure 1-1) which effectively reduce the stiffness of the overall medium.

3. a combination of the above two factors and possibly other undefined factors may be involved.

It is well known that field moduli are lower than laboratory determined values. The decrease in modulus of a rock as discontinuities are introduced, also known as size effect, has been documented, for example, by Heuze et al. (1981) who show that the field modulus can be as little as 10% of the intact rock modulus. However, even a tenfold decrease in modulus is not sufficient to explain the difference between analytical and empirical results (a factor of 30-50 appears to be necessary). For this reason it appears likely that item 3 above is the best explanation. Certainly factors such as the opening of tensile cracks, bed separation, roof caving, all act in a manner that is equivalent to reduction in stiffness. It is apparent from the additional analyses of Section 5.4 that virtually any desired result can be obtained by varying input parameters appropriately. It is important that anyone wishing to model or predict subsidence must have a good understanding of the rock mass properties.

The very crude simulation of roof caving using GEOROC only resulted in a marginal increase in subsidence. This result, which agrees with engineering beam theory, suggests that roof caving may not be as significant a factor in subsidence as has generally been thought.

The preceding analyses suggest that mechanisms in addition to simple strata bending are involved. Since these appear to act in a way which reduce the overall stiffness, a very important parameter is the field modulus for the overburden. Certainly, it was possible to make the analytical methods match the empirical curves by adjusting the overburden modulus appropriately. This approach, in which the field modulus incorporates both size effect and subsidence mechanisms, deserves further consideration since it would allow the effective use of the many analytically based programs which are available and avoid the complexity of modeling the various mechanisms separately.

6 FUTURE TRENDS

It is difficult to predict the future trends in subsidence analysis. However, this section will explore some possible areas where evolution will take place and the ramifications of these developments.

It is clear that the current vigorous growth in computer power will continue unabated to at least the end of the decade, and probably to the end of the century. For this reason numerical techniques will become increasingly more attractive tools for subsidence prediction. In particular, the finite element method will undoubtedly continue to evolve and become more and more sophisticated with many specialized non-linear analysis capabilities. At the same time computers are improving in speed and size and it is quite probable that in the not-too-distant future it will be possible to model subsidence and explicitly include all the mechanisms described in Table 1-1(b). As computing costs decrease three-dimensional FEM analyses will also become more standard. In addition, interactive programs for creating input files should become standard and will greatly simplify what presently is a very onerous and time-consuming task. It appears that the application of the finite difference method to subsidence predictions will remain at a low level. It is more difficult to extend boundary integral theory to include non-linear behavior and complex rock properties and progress in this area will probably not be as rapid as for the FEM.

It is foreseen that the application of hybrid techniques such as described in Section 2.7 will become more popular and will also become more sophisticated. The hybrid method is very powerful since it combines the empirical method and a mathematical description of the subsidence mechanism. The application of statistical procedures provides quantitative methods for determining parameters for evaluating the functional form of the mechanism and, thus, giving a better understanding of the subsidence process.

The stochastic models which were described in Section 2.6 will probably not receive research attention in the future. However, the application of statistics and probability functions has good potential for subsidence research in a different approach. That is, rock properties can be assigned probability distributions instead of definite values. These probability distributions will reflect the uncertainty in the value of that particular

rock property as well as its natural variation in situ. In addition, if there are uncertainties as to which subsidence mechanisms are dominant, they can be assigned probability distributions which reflect this uncertainty. This methodology coupled with, say, the FEM, will result in subsidence predictions with which statistical levels of confidence and ranges of variation can be associated. This approach has been used in groundwater flow in fractured rock where it is impossible to exactly locate each individual fracture by measurements (Andersson et al., 1984). In a similar vein, a systems variability analysis technique has been developed to calculate the radiological impact of a nuclear waste disposal system, taking into account uncertainties in the values of parameters that are involved. These parameters are defined by probability distributions rather than single values (Dormuth and Sherman, 1981). The application of the methodologies developed in these other fields to subsidence will be a promising area of development.

All of the methods described here for possible future trends in subsidence prediction involve specialized knowledge in such areas as numerical analysis, statistics, and continuum mechanics. For this reason the developments will undoubtedly be made by research organizations, and it will be a major challenge to transfer the technology so that it is readily accessible and can easily be used by mining engineers at their mine sites.

An important concern is that the capability to perform subsidence analyses is increasing much faster than the ability to acquire the rock mass properties needed as input for these analyses. While computer costs continue to decline, the costs of drilling boreholes, running geophysical surveys, and testing rocks in the laboratory have all been increasing steadily. At the present time the costs for characterizing a geologic section in the detail that would be required for a rigorous subsidence analysis are prohibitive. A major breakthrough in the data acquisition field is not expected in the next decade.

7 CONCLUSIONS AND RECOMMENDATIONS

This report presents a review and assessment of the empirical and analytical methods which can be used to predict surface subsidence over underground coal mining. A summary of accessible and documented computer programs for subsidence prediction was undertaken and six specific programs were selected and used to analyze a hypothetical case history which is typical of the Sydney coalfield. The results indicate that there is a major difference between subsidence values obtained by empirical methods and those produced by analytical means, with the latter yielding lower vertical displacements than the former method. An assessment of future trends indicates that the technical capability of subsidence analysis will increase over the next decade in parallel with the fast growth in computing power.

To date, there are no analytical methods which can, a priori, reliably predict subsidence. The successful prediction of subsidence is exceedingly complex and, indeed, is one of the most difficult and challenging problems in modern rock mechanics. This study suggests that subsidence is not simply a matter of elastic strata bending but involves a number of other complex mechanisms as well, (see Figure 1-1 and Table 1-1), which span the entire coal seam and overburden stratigraphy. The limited analyses which were performed as part of this study suggest that:

(1) roof caving is probably not a major contributor to total subsidence.
(2) laboratory determined elastic moduli are not representative of in-situ moduli. In-situ moduli are probably much smaller than the corresponding laboratory elastic moduli values.
(3) the 'field deformation modulus' is a useful parameter in subsidence which may be able to characterize both the effects of scale and some of the mechanisms involved.

To improve the state of the art in subsidence, progress in two fundamental areas is required. The first is to identify the mechanisms which are involved in the subsidence process, and to determine the importance of each to the overall process. It will also be necessary to formulate methods for modeling these mechanisms either explicitly (by forming the appropriate

constitutive equations) or implicitly (for example, by incorporating the effect of the mechanism into an appropriately adjusted field deformation modulus).

The second area is to improve the data base of field observations of subsidence. The data base in North America, especially in Canada, is inadequate and no serious progress in subsidence prediction can be made until this situation is rectified.

A number of recommendations are listed below. These address not only the problem of improving the state of the art in subsidence in general but also the specific problem of subsidence prediction for the mines in the Sydney coalfield. It is recommended that:

(1) a long-term program be initiated to establish a unified and comprehensive data base for subsidence field measurements. This program might be undertaken jointly by CANMET and the U.S. Bureau of Mines and would set standards for subsidence measurement, periodically collect and compile data, and manage and interpret the large data file of observations. Data should include not only field measurements of surface displacements but also geological descriptions, rock properties and any other useful information. This data base will form the foundation for research into the fundamental understanding of subsidence and also provide a solid basis for empirical and hybrid prediction methods.

(2) programs be established to study the various mechanisms involved in the subsidence process. This could take place through field experiments (e.g. long extensometers to study bed separation) and/or by back-analysis of existing case histories. Back analysis can also be used to relate laboratory determined values of rock properties to the in-situ rock mass properties. Clearly, a large collection of case history data is required to pursue the latter option.

(3) that a study be made of subsidence case histories in eastern and north-eastern U.S.A. These case histories can be used to calibrate, for example, the zone factors in the empirical zone method which can be applied to the Sydney coalfield. Cape Breton lies in the same tectonic unit, the Appalachian Mountain Belt, in which considerable coal mining is done in the U.S.A., and it should be possible to find a site with comparable geologic conditions.

(4) the Cape Breton Coal Research Laboratories acquire a few of the subsidence computer programs described in Section 4 and perform some analyses of the mines. It is recommended that more than one program be acquired since no single method is currently capable of reliably predicting subsidence. One (or more) program(s) might be chosen for ease of application, while others might be selected for research purposes. By actually analyzing the mine situation(s), guidance can be provided to the instrumentation development and deployment segments of the overall CANMET subsidence research program.

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APPENDIX

USER'S MANUAL AND LISTING OF PROGRAM BERRY.3D

INTRODUCTION

BERRY.3D is an analytical subsidence prediction program that calculates ground surface displacements in the x, y and z directions and horizontal strain in the x and y directions. The program uses a three-dimensional model of a homogeneous, linearly elastic and transversely isotropic rock mass. It is based on the closed-form solution developed by Berry and Sales (1962). The modified solution by Avasthi and Harloff (1982) which empirically simulates the U.K. National Coal Board results has also been included. Users who are interested in the method of solution may wish to consult the above-mentioned references.

ASSUMPTIONS

The analysis assumes that:

1. The ground remains intact and deforms elastically.
2. The mined seam is completely closed after extraction.
3. The depth of excavation is large compared with its width.
4. The (vertical) thickness of the excavated panel is small compared to its depth and (horizontal) width and length.

COORDINATE SYSTEM AND ELASTIC CONSTANTS

The origin of the x, y and z Cartesian coordinate system is on the earth's surface and is located at the centre of the rectangular plan view of the mined panels. Positive z is vertically upwards and the medium is therefore the semi-infinite region $z \leq o$.

The rock mass is assumed to be homogeneous, linearly elastic and transversely isotropic. The elastic constants K_1 and K_2 are related to the elastic properties perpendicular and parallel to bedding (horizontal planes). They are defined as:

$$K_1^2 = \frac{1 - \nu_x^2}{E_x/E_z - \nu_z^2} \qquad (1)$$

$$K_2 = \frac{E_x/2G - V_z\,(1 + V_x)}{E_x/E_z - V_z^2} \qquad (2)$$

where:

E_x, V_x = Young's modulus and Poisson's ratio in the horizontal direction.

E_z, V_z = Young's modulus and Poisson's ratio in the vertical direction.

G = Shear modulus in the vertical plane.

These two constants are usually obtained by a curve-fitting procedure from the existing subsidence data. The program is developed only for the case where K_1 K_2. For isotropic media $K_1 = K_2 = 1$.

INPUT AND OUTPUT

BERRY.3D has been written in BASIC 2.0 for interactive use on Hewlett-Packard series 200 microcomputers. The user is queried for input in relatively straight-forward English and there is no requirement to learn any variable names.

The user must enter the following information:

1. Title for the problem.
2. Type of analysis (analytical or empirical).
3. Initial x coordinate for output points.
   Step size in x direction.
   Number of points required in x direction.
4. Initial y coordinate for output points.
   Step size in y direction.
   Number of points required in y direction.
5. Elastic constants K_1 and K_2.
6. Half width of the panel (parallel to x axis).
   Half length of the panel (parallel to y axis).
   Depth to centre of panel.
   Panel thickness.

BERRY.3D calculates the induced displacements on the earth's surface only. The displacements in three directions (x, y and z) are positive in the positive coordinate directions. Horizontal strains are calculated in both x and y directions. A positive value indicates tensile (extension) strain and negative denotes compressive (shortening) strain. It should be noted that the accuracy of strain calaulation depends on the step size specified by the user (smaller step size means more accurate values).

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```
1     ! TITLE: BERRY3D
2     ! DATE:  FEB, 1985
3     ! BY:    A. POON
4     ! CONTENT:
5     !         THIS PROGRAM CALCULATES VERTICAL DISPLACEMENT AT
6     !         ANY POINT (X,Y) ON THE GROUND SURFACE. ALL
7     !         EQUATIONS ARE GIVEN IN THE PAPER "AN ELASTIC
8     !         TREATMENT OF GROUND MOVEMENT DUE TO MINING-III",
9     !         BY BERRY AND SALES(1962).
10    !         THE MODIFIED VERSION IS BASED ON THE PAPER "SUBSIDENCE
11    !         ASSOCIATED WITH SINGLE AND MULTICAVITIES FOR UNDER
12    !         GROUND COAL GASIFICATION" BY AVASTHI AND HARLOFF(1982),
13    !         PUBLISHED BY GULF SCIENCE AND TECHNOLOGY COMPANY,
14    !         IN THE JOURNAL OF ENERGY RESOURCES TECHNOLOGY,
15    !         JUNE 1982, VOL.104, PP.99-104.
17    OPTION BASE 1
18    !
19    !
20    !  DEFINITION OF VARIABLES
21    !    AU:  DISPLACEMENT IN THE X-DIRECTION
22    !    AV:  DISPLACEMENT IN THE Y-DIRECTION
23    !    AW:  DISPLACEMENT IN THE Z-DIRECTION
24    !    AX:  X-COORDINATE
25    !    AY:  Y-COORDINATE
26    !  XSTR:  STRAIN IN THE X-DIRECTION
27    !  YSTR:  STRAIN IN THE Y-DIRECTION
28    !   AXS:  MEAN X-COORDINATE
29    !   AYS:  MEAN Y-COORDINATE
30    !FIRSTX:  INITIAL X-COORDINATE
31    !FIRSTY:  INITIAL Y-COORDINATE
32    !INCR-X:  INCREMENTAL STEP SIZE IN THE X-DIRECTION
33    !INCR-Y:  INCREMENTAL STEP SIZE IN THE Y-DIRECTION
34    ! NUM-X:  NUMBER OF POINTS REQUIRED IN THE X-DIRECTION
35    ! NUM-Y:  NUMBER OF POINTS REQUIRED IN THE Y-DIRECTION
36    !
37    !
39    REAL R(2,2),Sy(41),Sx(4)
40    REAL A(4),B(4),C(4),D(4)
41    REAL Ax(41,41),Ay(41,41),Aw(41,41),Au(41,41),Av(41,41)
42    REAL Atk(4,2,2)
43    REAL Axs(41,41),Ays(41,41),Xstr(41,41),Ystr(41,41)
44    DIM Title$[80]
45    CALL Titlepage
46    INPUT "ENTER THE TITLE FOR THE PROBLEM",Title$
47    INPUT "ENTER TYPE OF ANALYSIS: BERRY=1 AND MODIFIED BERRY=2",Opt
50    INPUT "ENTER  FIRSTX,INCR-X AND NUM-X",X,Dx,Nx
50    INPUT "ENTER  FIRSTY,INCR-Y AND NUM-Y",Y,Dy,Ny
51    !
62    INPUT "ENTER ELASTIC PROPERTIES K1 AND K2",K1,K2
63    INPUT "ENTER 1/2 SEAM WIDTH,1/2 LENGTH,DEPTH AND THICKNESS",Sw,Sl,H,T
64 Echo:   !
67    PRINT Title$
68    PRINT
69    PRINT
71    IF Opt=1 THEN PRINT "TYPE OF ANALYSIS: BERRY"
72    IF Opt=2 THEN PRINT "TYPE OF ANALYSIS: MODIFIED BERRY"
73    PRINT
74    PRINT "FIRSTX  FIRSTY       DX         DY         NX         NY "
75    PRINT X,Y,Dx,Dy,Nx,Ny
```

```
 76     PRINT
 77     PRINT "ELASTIC PROPERTIES K1 AND K2 ARE:"
 78     PRINT K1,K2
 79     PRINT
 80     PRINT "SEAM GEOMETRY:"
 81     PRINT "        1/2 WIDTH  1/2 LENGTH  DEPTH  THICKNESS"
 82     PRINT " ",Sw,Sl,H,T
 83     PRINT
 84     INPUT "DO YOU WANT A HARDCOPY LISTING....Y/N ?...PRESS N TO CONTINUE",Ab$
 85     IF Ab$="Y" THEN
 86     PRINTER IS 701
 87     GOTO Echo
 88     END IF
 90     PRINTER IS 1
 92     PRINT
 94     PRINT "CALCULATON IN PROGRESS............."
 95     ! INITIAL FACTOR IS 1
 96     Factor=1.
 97     !
 98     IF Opt=1 THEN GOTO Endm
 99     Factor=1.
100     Factow=1.
101     Factol=1.
102     IF Sw/H<.3 THEN Factow=.621+5.011368*(Sw/H)^2.1445069
103     IF Sl/H<.3 THEN Factol=.621+5.011368*(Sl/H)^2.1445069
104     Factor=Factor*Factow*Factol
105 Endm: ! THIS IS END OF USING MODIFICATION FACTORS
106     !
107     !    INITIALIZE ALL STRAIN ARRAYS
108     FOR N2=1 TO Ny
109     FOR N1=1 TO Nx
110     Xstr(N1,N2)=0
111     Ystr(N1,N2)=0
112     NEXT N1
113     NEXT N2
114     !
115     !
116     Fx=X
117     FOR N2=1 TO Ny
118     X=Fx
119     FOR N1=1 TO Nx
120     !
121     !  EMPIRICAL TRANSFORMATION REQUIRED FOR THE SOLUTION
122     !  GIVEN BY AVASTHI AND HARLOFF (1982)
124     !
125     IF Opt=2 THEN X=X*2.
126     IF Opt=2 THEN Y=Y*2.
127     R(1,1)=Sw-X
128     R(1,2)=Sw+X
129     R(2,1)=Sl-Y
130     R(2,2)=Sl+Y
131          Sh=H^2
132      Sy(1)=(R(1,1)^2+R(2,1)^2)/Sh
140      Sy(2)=(R(1,2)^2+R(2,1)^2)/Sh
150      Sy(3)=(R(1,2)^2+R(2,2)^2)/Sh
160      Sy(4)=(R(1,1)^2+R(2,2)^2)/Sh
170     !
180      Sx(1)=R(1,1)*R(2,1)/Sh
190      Sx(2)=R(1,2)*R(2,1)/Sh
200      Sx(3)=R(1,2)*R(2,2)/Sh
```

```
210     Sx(4)=R(1,1)*R(2,2)/Sh
220     !
230     FOR N=1 TO 4
240     A(N)=Sy(N)+K2/(K1^2)
250     B(N)=(A(N)^2+(K1^2-K2^2)/K1^4)^.5
260     C(N)=(2*(K1*B(N)-K2*A(N)-(K2/K1)^2+1))^.5
270     D(N)=(4*K1*B(N)-C(N)^2)^.5
280     NEXT N
290     !
300     Sum1=0.
310     Sum2=0.
320     FOR N=1 TO 4
330     D1=B(N)+C(N)*Sx(N)+K1*Sx(N)^2
340     D2=B(N)-C(N)*Sx(N)+K1*Sx(N)^2
350     Dq1=LOG(D1/D2)
360     Sum1=Sum1+Dq1
370     !
380     D3=D(N)*Sx(N)
390     D4=B(N)-K1*Sx(N)^2
400     Dq2=ATN(D3/D4)
402     IF D4<0. THEN Dq2=Dq2+PI
405     IF Dq2>PI THEN Dq2=Dq2-PI*2.
410     Sum2=Sum2+Dq2
420     NEXT N
430     !
431     !  CALCULATE ELASTIC VERTICAL DISPLACEMENT
432     !
440     Ckk=(K1+K2)/(K1-K2)
450     W=-(.5*Ckk^.5*Sum1+Sum2)/(4*PI)*T
451     W=W*Factor
453     Aw(N1,N2)=W
454     !
455     ! A SIGN CHANGE IS REQUIRED IN THE FOLLOWING
456     ! FORMULATION BASED ON AVASTHI AND HARLOFF
460     !
461     Sy(1)=(R(1,1)^2+R(2,1)^2)/Sh
462     Sy(2)=(R(1,2)^2+R(2,2)^2)/Sh
463     Sy(3)=(R(1,2)^2+R(2,2)^2)/Sh
464     Sy(4)=(R(1,1)^2+R(2,1)^2)/Sh
465     !
466     FOR N=1 TO 4
467     A(N)=Sy(N)+K2/(K1^2)
468     B(N)=(A(N)^2+(K1^2-K2^2)/K1^4)^.5
469     NEXT N
470     !
472     Rth=(2^.5)/H
480     FOR N=1 TO 4
490     FOR I=1 TO 2
500     FOR J=1 TO 2
510     !
520     IF N<3 AND J=2 THEN GOTO 570
530     IF N>2 AND J=1 THEN GOTO 570
540     D5=2*(-(B(N)+A(N))^.5+(-1)^(J-1)*Rth*R(I,J))*((B(N)-A(N))^.5)
550     D6=(-(B(N)+A(N))^.5+(-1)^(J-1)*Rth*R(I,J))^2-(B(N)-A(N))
551     D7=ATN(D5/D6)
552     IF D6<0. THEN D7=D7+PI*2.
553     IF D7>PI THEN D7=D7-PI*3.
560     Atk(N,I,J)=D7
570     NEXT J
580     NEXT I
```

```
590    NEXT N
600    !
601       CALCULATE X AND Y HORIZONTAL DISPLACEMENT
602    !
610    Rsk=((N1-1.2)/2.)^.5
620    U=1/4./PI/Rsk*(Atk(1,2,1)-Atk(2,2,1)+Atk(3,2,2)-Atk(4,2,2))*T
630    V=1/4./PI/Rsk*(Atk(1,1,1)-Atk(2,1,1)+Atk(3,1,2)-Atk(4,1,2))*T
631    U=U*Factor
632    V=V*Factor
634    Au(N1,N2)=U
635    Av(N1,N2)=V
636    IF Opt=2 THEN X=X/2.
637    IF Opt=2 THEN Y=Y/2.
639    Ax(N1,N2)=X
640    Ay(N1,N2)=Y
641    !
644    X=X+Dx
645    NEXT N1
648    Y=Y+Dy
649    NEXT N2
650    !
651    PRINT CHR$(12)
653    PRINT "FACTOR=",Factor
654    PRINT
655    PRINT
656    PRINT "RESULTS OF LATEST ANALYSIS ON DISPLACEMENT VECTORS:"
657    PRINT
658    PRINT "X-COORD.    Y-COORD.    Z-DISPL.      X-DISPL.      Y-DISPL."
660    FOR N2=1 TO Ny
670    FOR N1=1 TO Nx
680    PRINT USING Fmt;Ax(N1,N2),Ay(N1,N2),Aw(N1,N2),Au(N1,N2),Av(N1,N2)
691 Fmt:    IMAGE 2(DDDD.DD,4X),3(DDD.DDDD,5X)
683    NEXT N1
684    NEXT N2
685    !
686    INPUT "DO YOU WANT A HARDCOPY LISTING....Y/N ?...ENTER N TO CONTINUE",A$
687    IF A$="Y" THEN
688    PRINTER IS 701
689    GOTO 651
690    END IF
691    PRINTER IS 1
692    !
693    !
694    ! THE FOLLOWING STRAIN CALCULATIONS ARE APPROXIMATE DEPENDING
695    ! ON THE WIDTH OF EACH INCREMENT
696    ! X-STRAIN IS THE SLOPE (DERIVATIVE) OF U-DISPL WRT X
697    ! Y-STRAIN IS THE SLOPE (DERIVATIVE) OF V-DISPL WRT Y
698    ! THE DERIVATIVE IS OBTAINED BY DIVIDING THE CHANGE IN
699    ! DISPLACEMENT BY DELTA X OR DELTA Y
700
701    !
703    FOR N2=1 TO Ny
704    FOR N1=1 TO Nx
705    IF Dx=0 THEN GOTO Abc
706    Xstr(N1,N2)=(Au(N1+1,N2)-Au(N1,N2))/Dx
707    IF Opt=2 THEN Xstr(N1,N2)=(Au(N1+1,N2)-Au(N1,N2))/(2*Dx)
708 Abc:  !
709    IF Dy=0 THEN GOTO 2nd
710    Ystr(N1,N2)=(Av(N1,N2+1)-Av(N1,N2))/Dy
711    IF Opt=2 THEN Ystr(N1,N2)=(Av(N1,N2+1)-Av(N1,N2))/(2*Dy)
```

```
712 Bcd:  !
714   NEXT N1
715   NEXT N2
716   FOR N2=1 TO Ny
717   FOR N1=1 TO Nx
718   Axs(N1,N2)=Ax(N1,N2)+Dx/2
719   Ays(N1,N2)=Ay(N1,N2)+Dy/2
720   NEXT N1
721   NEXT N2
722 Xz:  !
723   PRINT CHR$(12)
725   PRINT
726   PRINT "RESULTS OF LATEST ANALYSIS ON SURFACE STRAINS:"
727   PRINT
728   PRINT "X-COORD.    Y-COORD.     X-STRAIN      Y-STRAIN"
729   N2=1
730 Yz:  !
731   N1=1
732 Xy:  !
733   PRINT USING Fmt2;Axs(N1,N2),Ays(N1,N2),Xstr(N1,N2),Ystr(N1,N2)
734 Fmt2:    IMAGE 2(DDDD.DD,4X),2(DD.DDDDDD,4X)
735   N1=N1+1
736   IF N1<Nx THEN GOTO Xy
737   N2=N2+1
738   IF N2<Ny THEN GOTO Yz
739   !
740   INPUT "DO YOU WANT A HARDCOPY LISTING....Y/N ?...ENTER N FOR EXIT",Ab$
741   IF Ab$="Y" THEN
742   PRINTER IS 701
743   GOTO Xz
744   END IF
745   PRINTER IS 1
746   END
756   !
766   SUB Titlepage
776   PRINT CHR$(12)
786   GINIT
796   GRAPHICS ON
806   FRAME
816   MOVE 2,2
826   DRAW 131,2
836   DRAW 131,98
846   CSIZE 12,.6
856   MOVE 36,80
866   LABEL "BERRY3D"
876   PRINT USING "6/"
886   PRINT
896   PRINT "      AN ELASTIC THREE DIMENSIONAL MODEL FOR SUBSIDENCE PREDICTION"
906   PRINT "      OF TRANSVERSELY ISOTROPIC GROUND DUE TO MINING"
917   PRINT
926   PRINT "      PROGRAM WRITTEN BY A. POON (FEB. 1985) BASED ON SOLUTION"
936   PRINT "      GIVEN BY BERRY AND SALES(1962) AND AVASTHI AND HARLOFF(1982)"
946   PRINT
956   PRINT "      A PROPRIETARY PROGRAM OF RE/SPEC LTD., CALGARY, ALBERTA."
967   MOVE 113,17
978   CSIZE 6
989   LORG 7
1000  LABEL "R"
1011  MOVE 107.5,11.5
1022  DRAW 118.5,22.5
```

```
 963    LORG 3
 964    MOVE 113,17
 965    LABEL "S"
 966    R=3
 967    MOVE 113,17
 968    DEG
 969    RPLOT R,0,2
 970    FOR Th=0 TO 360
 971        X=R*COS(Th)
 972        Y=R*SIN(Th)
 973        RPLOT X,Y
 975      NEXT Th
 976    INPUT "          PRESS ENTER TO CONTINUE.......",Mm$
 978    GRAPHICS OFF
 979    PRINT CHR$(12)
 980    SUBEND
```